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Research on the Solution of Function and Derivative Questions in the National College Entrance Examination —Taking the National 1/2/3 Papers from 2018 to 2022 as an Example

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1. PROBLEM STATEMENT

The new college entrance examination has further increased the difficulty of mathematics, with the function and derivative section now carrying a higher weight in the exam. These concepts, characterized by their abstract nature, flexibility, and comprehensive scope, rigorously test students logical reasoning, intuitive visualization, and computational skills. As a persistent challenge in both student learning and teacher instruction, developing effective problem-solving strategies for functions and derivatives holds significant value for both high school students and educators.

2. RESEARCH METHODS

This study employs three primary research methodologies: literature analysis, content analysis, and statistical analysis. The literature analysis method provides a comprehensive overview of current research on functions and derivatives, establishing theoretical foundations and offering Polyas problem-solving theory as a strategic framework. The content analysis method examines the practical applications of functions and derivatives in high school mathematics, highlighting their instrumental characteristics to enhance problem-solving strategies. The statistical analysis method investigates the examination patterns of functions and derivatives in the National College Entrance Examination (Gaokao) from 2018 to 2022, including score distribution, question numbers, and total question volume in the national exam papers.

3. RESEARCH CONCLUSIONS

3.1 Analysis of the Question Number of Function and Derivative in the National College Entrance Examination

The mathematics section of the National College Entrance Examination (Gaokao) National Paper comprises three question types: multiple-choice, fill-in-the-blank, and essay questions. Each type carries distinct assessment priorities and scoring patterns, with difficulty levels increasing progressively from the first to the last question. Notably, functions and derivatives are consistently featured across all three question types. This analysis will examine the 2018-2022 National Papers (Volumes I, II, and III) from these three perspectives.

Table 1: Multiple-choice Question Numbers

Multiple-choice question number					
Category	2018	2019	2020	2021	2022
National Exam Paper 1	3, 7	3, 5, 11	5, 6, 7, 12	12	5, 6, 11, 12
National Exam Paper 2	4, 8, 10	6, 9, 12	3, 9, 11	4, 10, 12	1 2
National Three-Volume System	3, 7, 12	6, 7, 11	4, 10, 12	7,	6, 7, 10, 12

The table above displays the question numbers for function and derivative topics in the National College Entrance Examination (2018-2022). Analysis reveals these topics are primarily tested in questions 3-12, with questions 6-12 constituting the majority. The questions are predominantly medium to difficult in difficulty. The data indicates that the National Exam has progressively increased the difficulty level of function and derivative questions in

multiple-choice sections, as evidenced by their shifting to later question numbers over the years.

Table 2: Number of fill-in-the-blank questions

Fill-in-the-blank question number					
Category	2018	2019	2020	2021	2022
National Exam Paper 1	not have	13	not have	13	not have
National Exam Paper 2	15	14	not have	not have	15, 16
National Three-Volume System	14	not have	16	13, 15	15

The table above displays the function and derivative fill-in-the-blank question numbers in the National College Entrance Examination (NCEE) from 2018 to 2022. The data reveals that these topics are less frequently tested, with moderate difficulty levels and predominantly medium-difficulty questions. However, compared to other NCEE papers, Paper 3 demonstrates a stronger emphasis on function and derivative concepts, featuring two such questions annually except in 2019. This pattern indicates that Paper 3 prioritizes these mathematical concepts in its fill-in-the-blank section.

Table 3: Analysis Question Number

Analysis Question Number					
Category	2018	2019	2020	2021	2022
National Exam Paper 1	21	20	21	21	21
National Exam Paper 2	20	20	21	20	21
National Three-Volume System	21	20	21	22	22

The table above lists the analytical question numbers for functions and derivatives in the National College Entrance Examination (Gaokao) from 2018 to 2022. These questions consistently appear as the most challenging final problems, highlighting their pivotal role in gaokao and their function as the primary knowledge that differentiates student performance levels.

In summary, our analysis of function and derivative questions in the national college entrance exam over the past five years highlights their critical importance. These topics typically appear as the most challenging final questions, serving as a watershed moment for students. Function-related fill-in-the-blank questions maintain consistent difficulty levels, while multiple-choice questions have progressively shifted to later sections with increasing complexity. This progression underscores the paramount significance of mastering function and derivative problem-solving techniques.

3.2 Analysis of the Number and Score of Function and Derivative Questions in the National College Entrance Examination

The Gaokao (National College Entrance Examination) demonstrates the significance of different knowledge domains through varying question volumes, which also indirectly reflects their difficulty levels. Analysis of function and derivative question types over the past five years reveals that the 2021 and 2022 Gaokao introduced separate papers (A and B). For unified data analysis, the 2021 and 2022 National Gaokao papers are categorized as National Paper 1 (A) and National Paper 2 (B), respectively. This classification generates a line graph showing the distribution of multiple-choice, fill-in-the-blank, and analytical questions on function and derivative topics across National Papers 1, 2, and 3.

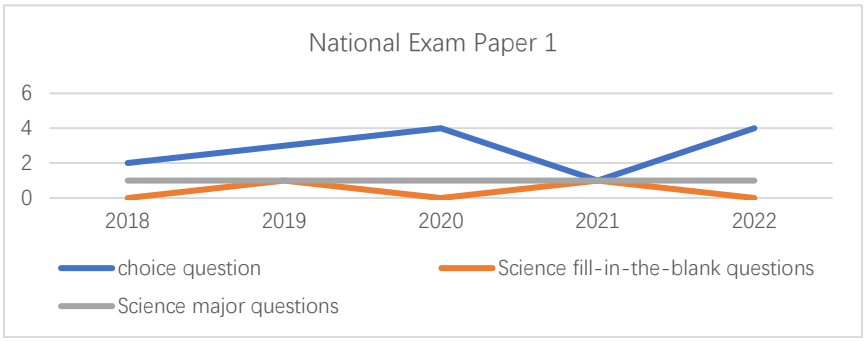

Figure 1: Question Volume Distribution for National Exam Paper 1

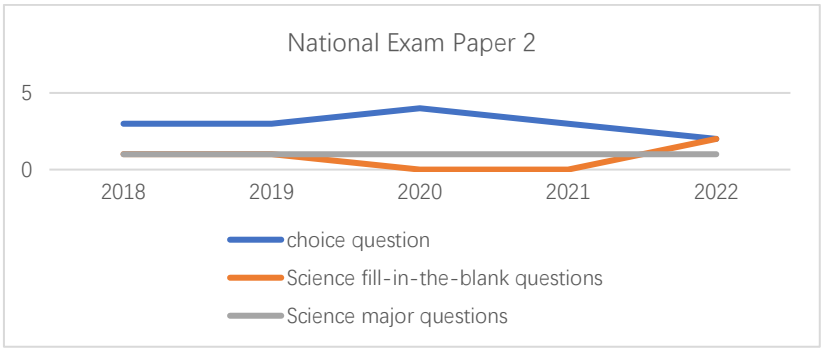

Figure 2: Question Volume Distribution for National Exam Paper 2

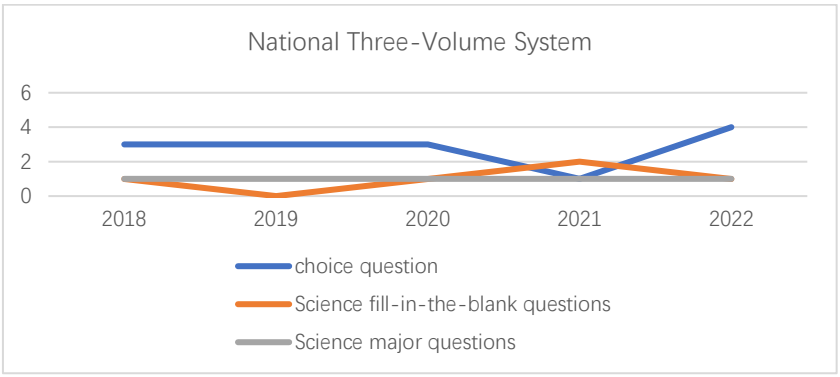

Figure 3: Question Volume Distribution for National Exam Paper 3

The national exam papers consistently feature a high volume of function and derivative questions. While the multiple-choice section in Paper 1 shows significant fluctuations, the overall trend remains upward, with fill-in-the-blank questions stabilizing. Paper 2 witnesses a decline in multiple-choice questions but an increase in fill-in-the-blank frequency. Paper 3 demonstrates a marked rise in multiple-choice questions. Three line charts reveal dramatic fluctuations in the number of these three question types across all three papers from 2021 to 2022, a phenomenon linked to the college entrance exam reforms. The growing number of function and derivative questions reflects their increasing importance, while also indicating a rise in difficulty.

The total score for the mathematics section of the National College Entrance Examination (Gaokao) is 1 50 points per paper annually, with varying weightings and fluctuations across different knowledge domains. These scores also reflect the relative difficulty of the questions. The table below shows the changes in the weight distribution of function and derivative questions across different papers each year.

Table 4: National test score distribution over the past five years

class	National Exam Paper 1	National Exam Paper 2	National Three-Volume System
2018	2 7	3 5	3 2
2019	3 2	2 7	3 2
2020	2 7	2 7	3 2
2021	3 2	3 7	2 7
2022	2 7	3 2	2 7

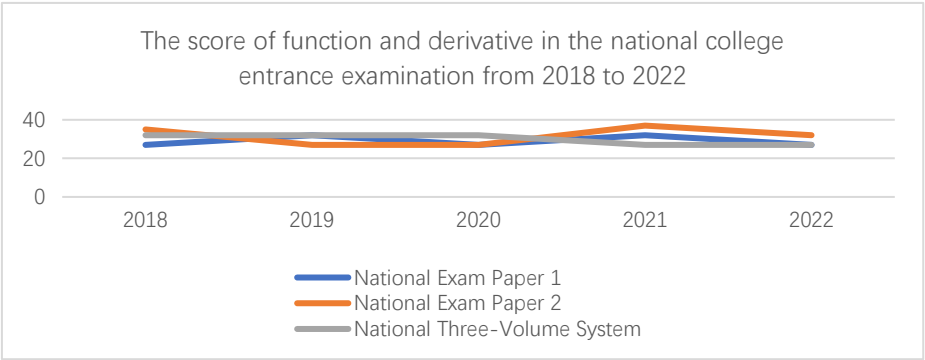

Figure 4: Score distribution of the National College Entrance Examination in the past five years

The weight of the functions and derivatives section in the National College Entrance Examination has shown fluctuating trends across different test papers. National Paper I exhibited a "one-year increase followed by one-year decrease" pattern. National Paper II saw a decline from 2018 to 2019, then experienced a significant rise from 2020 to 2021 before declining again. National Paper III remained relatively stable from 2018 to 2020, followed by a drop from 2020 to 2021. While National Paper II saw a substantial increase from 2020 to 2021, both National Paper I and other papers generally maintained stable performance with minimal fluctuations throughout the years. This trend indirectly indicates that the proportion of this section in the overall exam has stabilized.

3.3 Analysis of the Examination Content of Functions and Derivatives in the National College Entrance Examination

To analyze the key examination areas of functions and derivatives in Chinas National College Entrance Examination (Gaokao), this study identifies the primary assessment directions and focal points. Through systematic analysis, classification, and synthesis of real exam questions from 2018 to 2022, the research categorizes the content into five core components: 1) Function concepts and zeros, 2) Function properties, 3) Exponential and logarithmic functions, 4) Derivative concepts and their geometric interpretations, and 5) Related derivative applications. The five-year examination trends are systematically categorized as detailed in the table below.

Table 5: Examination Content of Functions and Derivatives in the National College Entrance Examination (Gaokao) over the Past Five Years

class	2018	2019	2020	2021	2022
Concept of Function, Zero Point	Questions 1 and 2 from National Paper 1 Question 3 from National Paper 2 Question 7, National Exam Paper 3 National Paper 3, Question 15	Question 9, National Exam Paper 2 Question 7, National Exam Paper 3	National Exam Paper 1, Question 1 0	Question 3 from National Exam Paper 2 National Exam Paper 2, Question 13 Question 7, National Exam Paper 2 National Exam Paper 2, Question 10 Question 5, National Exam Paper 3	National Paper 3, Question 15
properties of a function	National Exam Paper 2, Question 10 Question 11, Part 2 of the National Exam	Questions 1 and 2 from National Paper 2 Question 11, Part 3 of the National Exam	Question 8 from National Paper 1 Question 7, National Exam Paper 1 Question 9, National Exam Paper 2	Question 4, National Exam Paper 1 National Exam Paper 1, Questions 1 to 3 Question 8, National Exam Paper 2 Question 14 from National Paper 2 Question 4, National Exam Paper 1 Questions 1 and 2 from National Paper 3	Questions 1 and 2 from National Paper 1 Question 8 from National Paper 2 Questions 1 and 2 from National Paper 3
exponential and logarithmic functions	Question 9 from National Paper 1 Questions 1 and 2 from National Paper 3	Question 3 from National Paper 1 Question 6, National Exam Paper 2 Question 14, National Paper 2	National Exam Paper 1, Question 11 Questions 1 and 2 from National Paper 1 Question 11, Part 2 of the National Exam Questions 1 and 2 from National Paper 3	Question 7, National Exam Paper 1 Question 7, National Exam Paper 2	Question 8 from National Paper 1
The Concept of Derivative and Its Geometric Meaning	Question 5 from National Paper 1 National Exam Paper 2, Question 13 National Paper 3, Questions 1 to 4	Questions 1 and 2 from National Paper 1 National Exam Paper 2, Question 20 Question 6, National Exam Paper 3 Questions 1 and 2 from National Paper 3	National Exam Paper 3, Question 1 0 Question 6 from National Paper 1	National Exam Paper 1, Question 2 2 Question 1 6, National Paper 2 National Exam Paper 3, Question 1 3	Question 14 from National Paper 2 National Exam Paper 1, Question 14 Question 16, National Paper 3 Question 21, Part 2 of the National Exam
The Operation of Derivative and Its Application	Question 21, National Exam Paper 1 Question 21, Part 2 of the National Exam Question 21, Part 3 of the National Exam	National Exam Paper 1, Question 20 National Exam Paper 2, Question 20 National Exam Paper 3, Question 20	Question 21, National Exam Paper 1 Question 21, National Exam Paper 1 Question 21, Part 2 of the National Exam Question 21, Part 3 of the National Exam	National Exam Paper 2, Question 22 National Exam Paper 2, Question 20 Question 21, Part 3 of the National Exam National Exam Paper 1, Question 22	National Exam Paper 2, Question 22 National Exam Paper 1, Question 22

The analysis of the table reveals that in the 2018-2022 national exams, derivative operations and their applications consistently appeared as the final questions (questions 20, 21, and 22). Function concepts were primarily tested in 2018 and 2021, 41 while other derivative-related topics showed relatively stable annual distribution. Function properties received heightened attention in 2021, predominantly through multiple-choice and fill-in-the-blank questions. Derivative concepts and their geometric interpretations mainly appeared in analytical problems and fill-in-the-blank questions. Logarithmic and exponential functions were predominantly tested via multiple-choice questions, occasionally appearing as fill-in-the-blank questions. Function properties, similar to exponential and logarithmic functions, primarily used multiple-choice formats with occasional fill-in-the-blank questions. This analysis identifies key question formats for different topics, providing strategic guidance for students learning. Multiple-choice questions emphasize problem-solving techniques and speed, fill-in-the-blank questions focus on result accuracy, while analytical questions require comprehensive knowledge mastery and logical rigor.

The table below summarizes the number of questions on different examination topics across years. To provide a clearer visual representation of the proportion of each topic in different years, the line chart below has been created.

Table 6: Number of Function and Derivative Topics in the National College Entrance Examination (Gaokao) over the Past Five Years

	The Concept of Function and Zero Point	properties of a function	exponential function and logarithmic function	The Concept of Derivative and Its Geometric Meaning	The Operation of Derivative and Its Application
2018	4	2	2	3	3
2019	2	2	3	4	3
2020	1	3	4	2	4
2021	4	6	2	3	4
2022	1	3	1	4	2

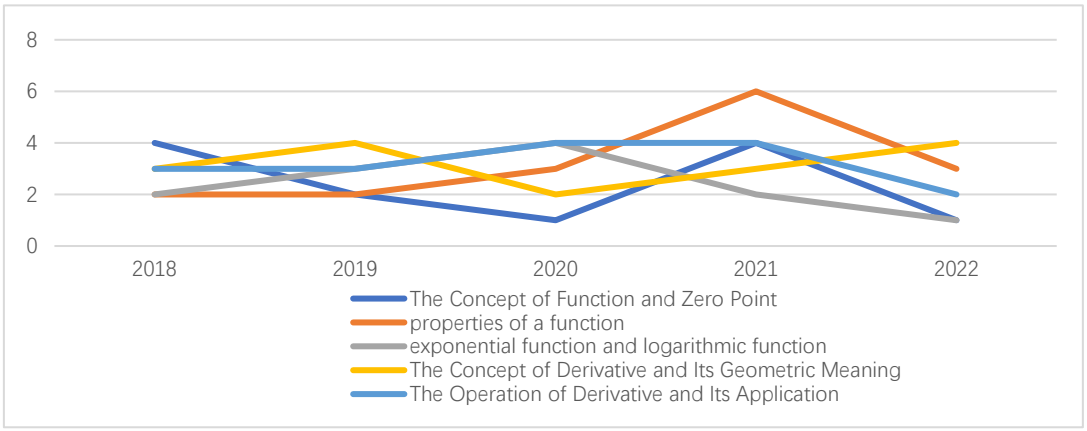

Figure 5: Line Chart of Function and Derivative Exam Topics in Chinas National College Entrance Examination (Gaokao) Over the Past Five Years

The table and line graph reveal that from 2018 to 2019, the concepts and geometric meanings of derivatives, exponential functions, and logarithmic functions maintained stability, while the operational applications of derivatives and function properties remained unchanged. However, the understanding of function concepts and zero points experienced significant decline. From 2019 to 2020, both derivatives and their geometric meanings declined, with function concepts and zero points declining again. Other examination topics showed increases. From 2020 to 2021, only exponential functions and logarithmic functions decreased, while other topics rose. From 2021 to 2022, derivatives and their geometric meanings increased, while others declined. Overall, the line graph indicates a substantial rise in function properties from 2020 to 2021. Over the five-year period, while there were fluctuations in the quantity of different examination topics, the overall trend remained stable. This suggests that future college entrance exam content will largely mirror the patterns observed in recent years.

3.4 Frequency Analysis of Different Types of Function and Derivative Questions in the National College Entrance Examination

In the previous section, we summarized and analyzed the examination content of functions and derivatives in the National College Entrance Examination (Gaokao). We then categorized and listed six common types of exam questions: zero-point problems, extremum problems, isomorphism problems, function construction, constant-identity problems, and inequality-solving problems. Through analyzing Gaokao questions from 2018 to 2022, we

identified the frequency of these six exam points, as shown in the table below.

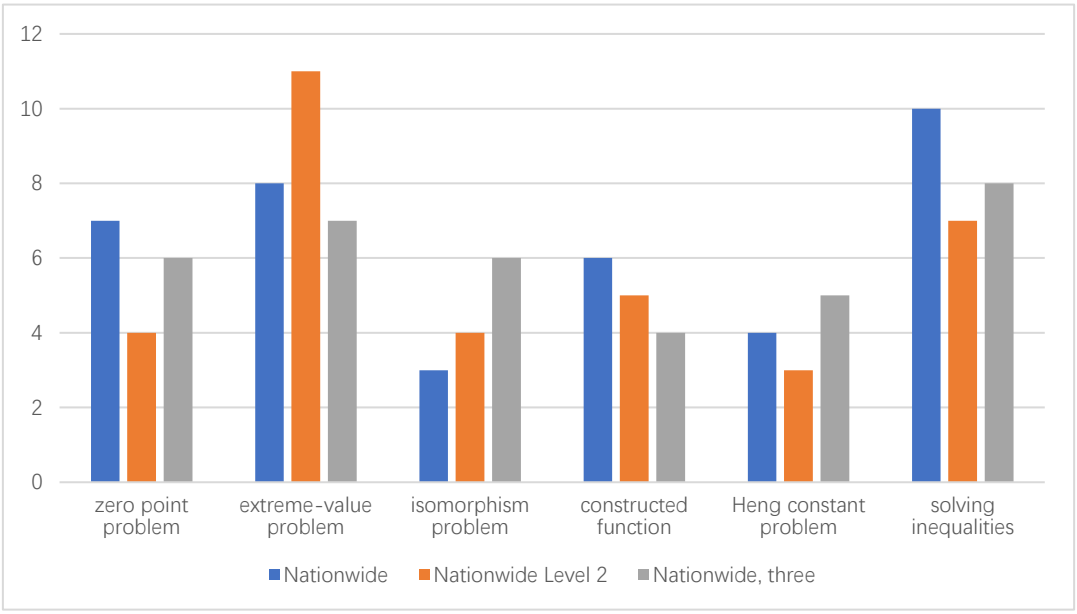

Figure 6: Five-Year Distribution of Key Exam Points in College Entrance Exam Functions and Derivatives (Column Chart)

As shown in the table, while the zero-point problem appears less frequently in National Exam II compared to other versions, extremum problems are more commonly tested in this paper. Regarding isomorphism problems, Papers I, II, and III show an increasing trend, whereas the construction function section demonstrates a decreasing pattern across all three papers. In terms of constant-identity problems, the number varies minimally between National Papers. However, inequality-solving and extremum problems consistently rank higher than other question types in total quantity. Specifically, National Exam II prioritizes extremum problems, National Exam I emphasizes inequality-solving, while National Exam III maintains a balanced distribution. Although variations exist in the quantity of different question types across papers and even within the same question type, six core problem categories—zero-point problems, extremum problems, isomorphism problems, construction functions, constant-identity problems, and inequality-solving—dominate the exam. These categories comprehensively cover all possible function and derivative question types in the National College Entrance Examination, with high frequency. Effective problem-solving strategies for these six categories remain crucial for tackling function and derivative challenges.

3.5 Difficulty Analysis of Functions and Derivatives

As the Gaokao reforms continue, the assessment of functions and derivatives has grown increasingly challenging. Through analyzing the difficulty of national science exam papers over the past five years, I adapted Bao Jianshengs model for evaluating comprehensive mathematical test difficulty, with modifications tailored to this studys context. The model categorizes difficulty into five dimensions: contextual factors, computational proficiency, reasoning ability, knowledge depth, and cognitive capacity, each assigned distinct weightings.

According to the five difficulty factors of the comprehensive difficulty coefficient model, they are assigned certain weight coefficients at different levels, and generally, the natural value is assigned to each level, that is, the natural number is assigned to the value. $1,2,3\ldots, using\ the\ formula d_i = \sum n_{ij} d_{ij} \sum n_{ij\,n}$, $i = 1,2,3\ldots)$ $to\ calculate\ the\ difficulty\ coefficients\ of\ each\ factor\ in\ the\ test\ paper, where$（$i$=1,2,3,4,5) $represent$ 5 $different\ dimensions n_{The\ ij\ represents\ the\ position\ of\ the\ element} the\ i-th\ dimension\ of\ thej\ levels\ of\ weights.$

Table 7: Difficulty Analysis of Function and Derivative Questions in National College Entrance Examination Papers over the Past Five Years

factor	horizontal	weight	Number of questions			percentage			Comprehensive difficulty coefficient		
			I roll up	II roll up	III roll up	I roll up	II roll up	III roll up	I roll up	II roll up	III roll up

background factor	No background	1	19	20	24	0.86	0.87	0.89	1.23	1.18	1.19
	Life Background	2	1	2	2	0.05	0.08	0.07			
	Scientific background	3	2	1	0	0.09	0.05	0			
	Integrated Background	4	0	0	1	0	0	0.04			
computational proficiency	Simple numerical operations	1	2	3	5	0.09	0.13	0.18	2.68	2.90	2.34
	complex numerical operations	2	9	11	12	0.41	0.47	0.45			
	Simple Symbol Operations	3	5	4	6	0.23	0.17	0.22			
	complex symbol computation	4	6	5	4	0.27	0.33	0.15			
inferential capability	Simple Reasoning	1	1	1	4	0.04	0.04	0.15	2.37	2.49	2.11
	general reasoning	2	12	10	16	0.55	0.43	0.59			
	complex reasoning	3	9	12	7	0.41	0.53	0.26			
Knowledge content	A knowledge point	1	1	2	5	0.04	0.08	0.19	2.28	2.35	2.13
	Two knowledge points	2	14	11	13	0.64	0.49	0.49			
	Three or more knowledge points	3	7	10	9	0.32	0.43	0.32			
Cognitive level	memorize	1	0	0	1	0.00	0	0.04	2.64	2.70	2.48
	utilize	2	8	7	12	0.36	0.30	0.44			
	aggregate analysis	3	14	16	14	0.64	0.70	0.52			

A statistical analysis was conducted on the proportion of five distinct factors in functions and derivatives across different proficiency levels during the past five years of the National College Entrance Examination (Gaokao). Corresponding difficulty coefficients for these factors in the national exam papers were also calculated. To enhance the visual clarity of the difficulty coefficient data and facilitate clearer comparisons of question volumes for the same factor at different proficiency levels, the following line charts presenting the data for each factor were created.

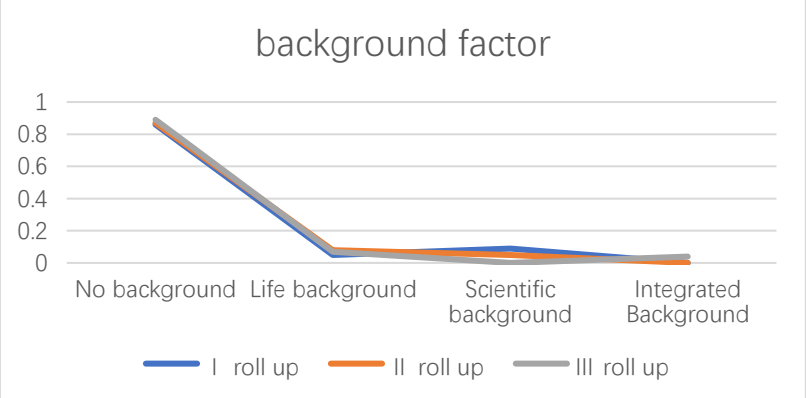

Figure 7: Line Chart of the Combined Difficulty of Background Factors

Background factors: The weight of four types of background factors—no background, life background, scientific background, and comprehensive background—increases in that order, with the national exam paper having the highest proportion of no background factors. Among the national exam papers, the first paper has the highest proportion of scientific background, followed by the second and third papers. The comprehensive background accounts for a negligible portion of the national exam papers, approaching zero in the past five years. In summary, function and derivative questions predominantly appear in no-background formats, with scientific background rarely encountered.

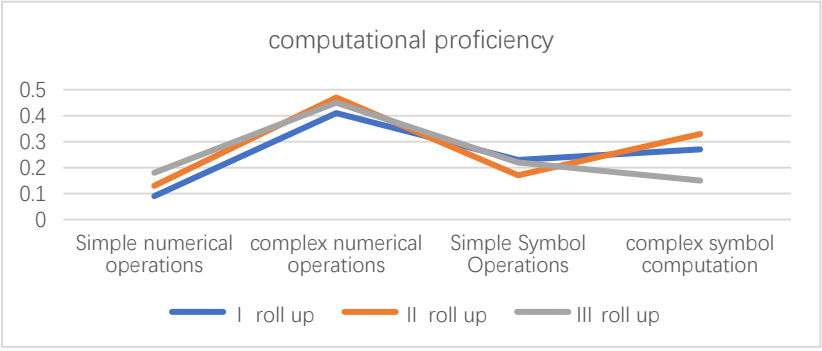

Figure 8: Line Chart of Comprehensive Difficulty Level

The computational proficiency levels—simple numerical operations, complex numerical operations, basic symbolic operations, and advanced symbolic operations—are assigned weights in ascending order of 1, 2, 3, and 4. The line graph reveals that the national exam maintains a consistent trend in computational proficiency levels. However, the National Exam II demonstrates a higher performance in advanced symbolic operations, accounting for approximately 30% of the total. Overall, complex numerical operations represent the highest proportion among the four categories, reaching around 45%. This indicates that the national exam primarily emphasizes complex numerical and symbolic operations, reflecting the higher difficulty of functions and derivatives.

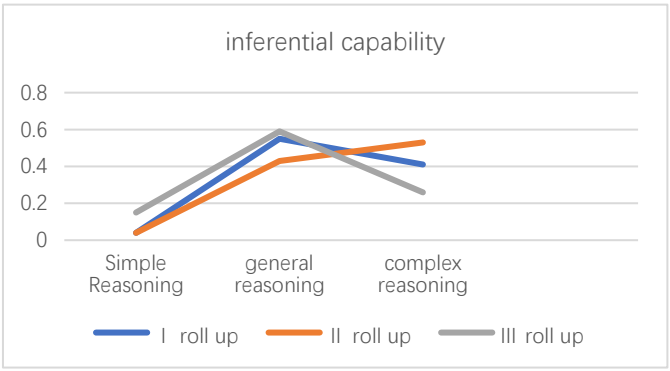

Figure 9: Line graph of comprehensive difficulty of reasoning ability

Reasoning ability: The trend of reasoning ability in the national exam is basically the same as the operation level, that is, in complex reasoning, the second national exam is more prominent, reaching 53%, and the general reasoning in the national exam is in the range of 40%-60%, accounting for the main part. However, the complex reasoning is only about 5% less than the general reasoning.

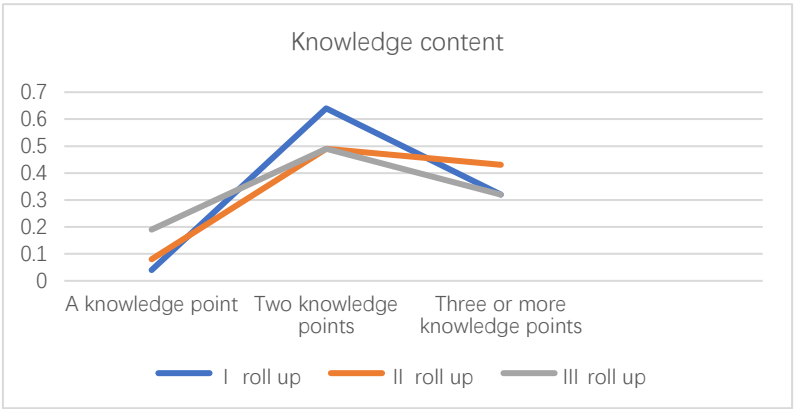

Figure 10: Knowledge Content Complexity Line Chart

Content Coverage: Analysis of the line chart reveals that the blue-colored National Exam Paper 1 demonstrates exceptional performance in two knowledge domains, achieving a peak coverage rate of 65%. While National Exam Papers 2 and 3 share the same 46% coverage rate, Paper 2 significantly outperforms others with 43% coverage across three or more domains. Notably, while both Papers 1 and 2 show negligible coverage (near zero) in a single domain, Paper 3 achieves a remarkable 20% coverage rate.

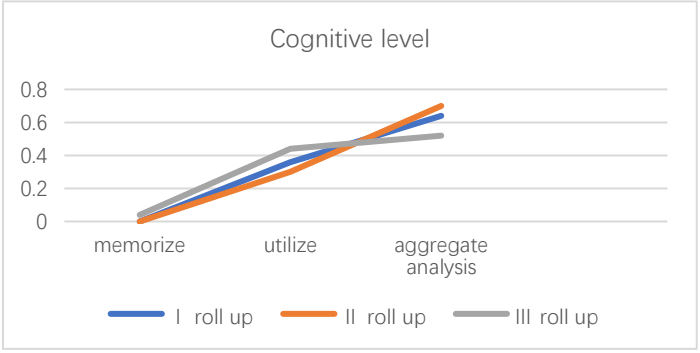

Figure 11: Line Chart of Comprehensive Difficulty Level of Cognitive Abilities

Cognitive Level: The national test paper shows a nearly straight line in its trend. A comprehensive analysis indicates that the national test paper accounts for 50%-70% of the total, while the second national test paper reaches as high as 70%. Memorization: Only the third national test paper accounts for 4%, with the other two being zero.

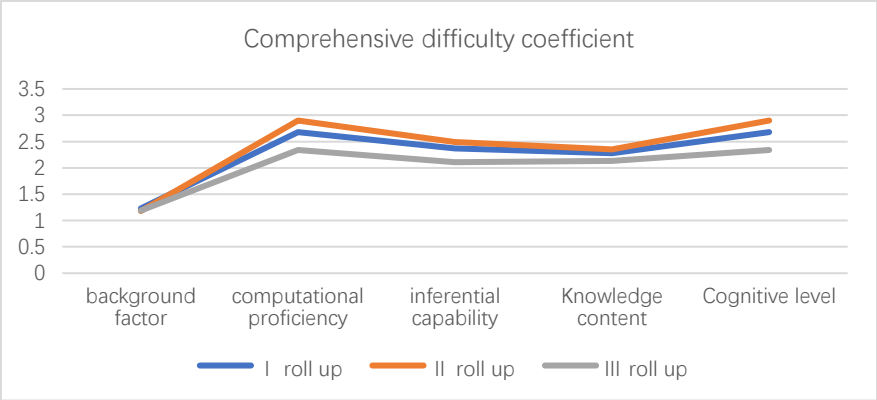

Figure 12: Combined Difficulty Line Chart

The National College Entrance Examination (NCEE) papers for functions and derivatives demonstrate alignment across five difficulty factors. Among these, computational proficiency ranks highest, followed by cognitive ability, reasoning capacity, knowledge depth, and contextual factors. Notably, the contextual factor difficulty consistently averages around 1.3 on the NCEE scale, showing a significant gap compared to other factors. This indicates that the function and derivative questions in the NCEE essentially exclude contextual elements entirely.

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