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Research on the Solution of Derivative Problems in the New College Entrance Examination Mathematics — Taking the 2020-2023 New College Entrance Examination Papers as an Example

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1. PROBLEM STATEMENT

As the college entrance examination reform progresses, the difficulty and requirements of mathematics test questions continue to rise. The emergence of new derivative questions in the college entrance exam reflects the direction of mathematics education reform and the evolving difficulty of exam questions. This research project aims to study these derivative questions to provide students with effective problem-solving methods, enhance their understanding of derivative concepts, help them achieve higher scores in the exam, and encourage exploration of derivative knowledge. It also seeks to offer valuable references for teachers in teaching this subject matter.

2. RESEARCH METHODS

2.1 Literature Research Method

According to the research subject, through consulting the relevant materials, the research content of the solution strategy of the derivative problem is understood, in order to master the solution strategy of the derivative in the mathematics examination of the national college entrance examination, and to facilitate further exploration.

2.2 Descriptive Research Method

The selected research topic explores the essence and characteristics of the subject through the authors interpretation. This paper analyzes and compares derivative problems in the 2020-2023 National College Entrance Examination (Gaokao) mathematics section, examining question types, quantity, and scoring criteria. The findings are presented in tabular form, followed by critical reflections and a summary.

2.3 The Method of Practical Research

This paper combines the students learning situation observed during the internship, and analyzes the students knowledge mastery and the teachers actual situation, and then puts them into the problem-solving strategy.

3. RESEARCH CONCLUSIONS

Derivatives carry a substantial weight in the National College Entrance Examination (Gaokao), with core concepts remaining largely consistent but significantly more challenging. Students should therefore tailor their approach: secure the foundational scores based on their capabilities, while developing systematic strategies to tackle the more complex sections and master the simpler components.

3.1 Derivative Questioning Form and Score Statistics of the New Gaokao

Table 1: Statistics of the Questioning Form of the New National College Entrance Examination from 2020 to 2023

examination paper	choice question quantity	gap filling quantity	subjective item quantity	total quantity	Number of questions proportion	Score percentage
I roll up	4	5	4	13	14.77%	15.50%
II roll up	4	3	4	11	12.50	13.8%
Beijing Volume	3	2	4	9	10.22%	12.16%
Tianjin Paper	2	0	4	6	6.81%	9.66%
Shanghai Paper	2	2	4	8	9.09%	11.33%
total quantity	15	12	20	47	\	\

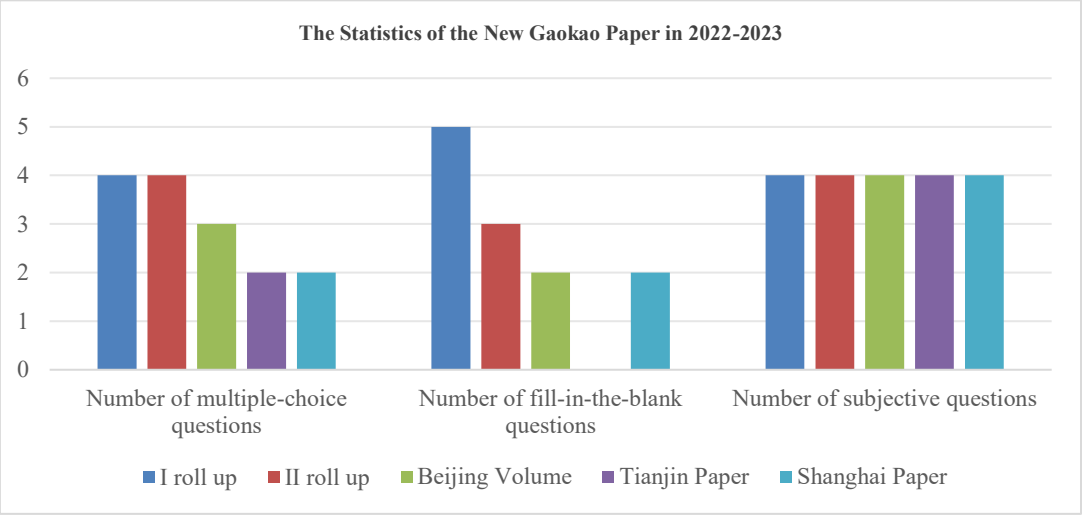
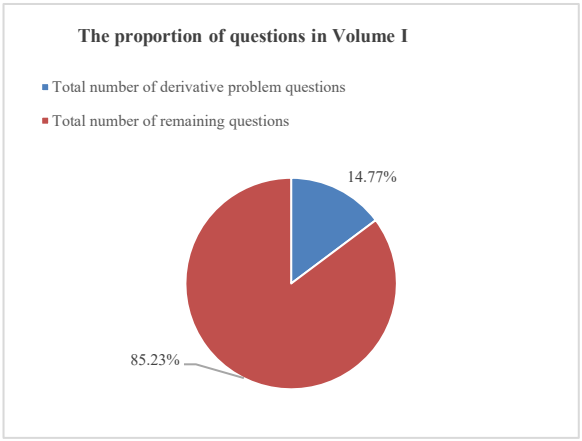
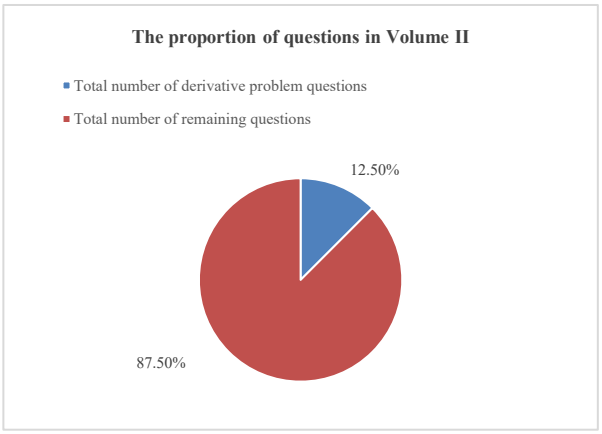


Figure 1: Statistics of the Questioning Form of the New National College Entrance Examination from 2020 to 2023

This study primarily examines derivative problems from the 2020-2023 National Mathematics I and II exams under Chinas new college entrance examination system, along with the 2 020 regional exams. It analyzes key examination points and problem-solving techniques, summarizing effective methods and strategies. As shown in Figure 2, derivative questions encompass multiple-choice, fill-in-the-blank, and subjective types, with their proportions remaining relatively consistent across years. The national exams typically include one multiple-choice question, one fill-in-the-blank question, and one subjective question in the derivative section. Regional exams, however, invariably feature one subjective question, while the number of multiple-choice and fill-in-the-blank questions varies significantly.





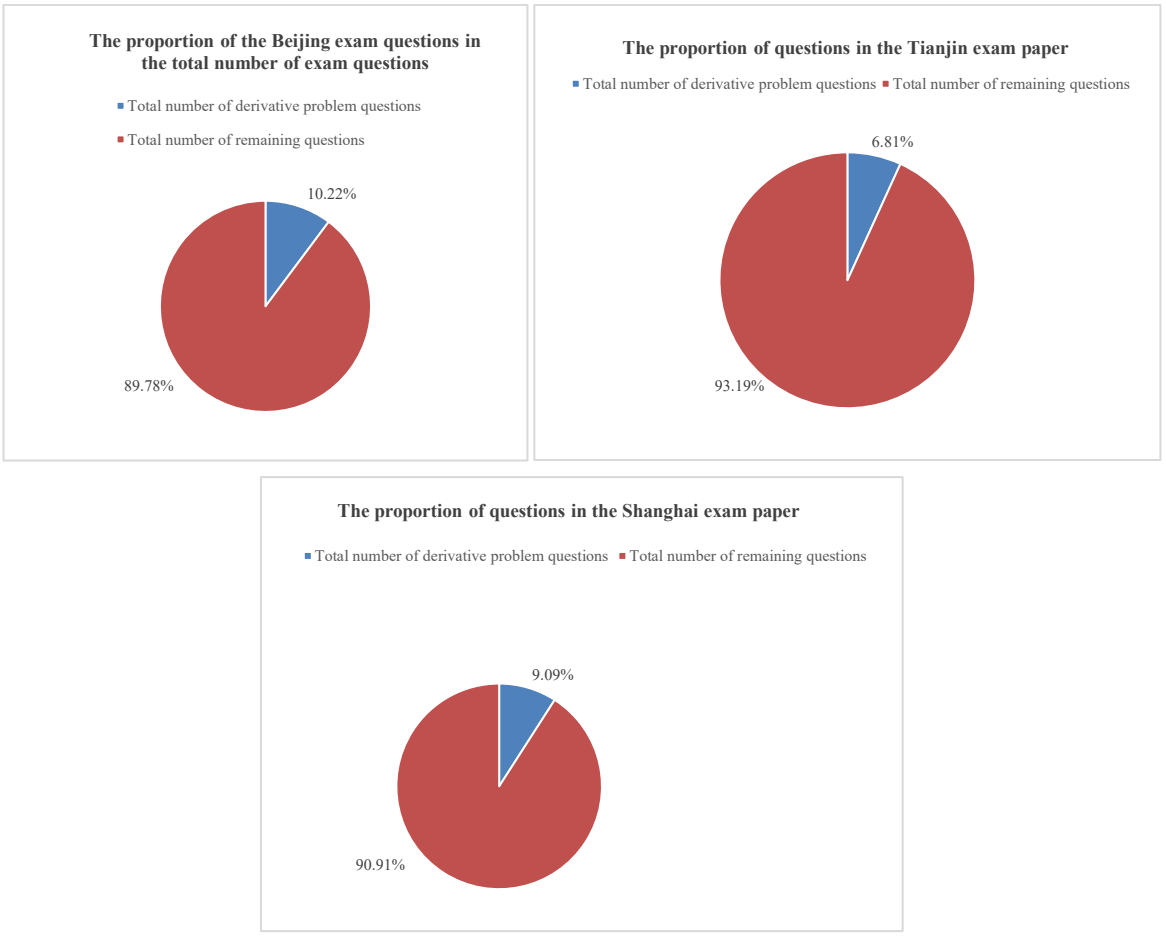


Figure 2: The proportion of questions on derivatives in the National College Entrance Examination (Gaokao) Papers I/II and local papers from 2020 to 2023

Analysis of Figure 3 reveals that in the new college entrance examination, the proportion of derivative questions is distributed as follows: Papers I/II account for 14.77% and 12.50% respectively, while the Beijing paper makes up 10.22%, the Tianjin paper 6.81%, and the Shanghai paper 9.09%. The number of derivative questions in Papers I/II remains relatively stable and substantial, whereas regional papers show greater variability and fewer questions. Students may therefore begin with subjective questions to familiarize themselves with derivative content, or conduct targeted research to develop problem-solving strategies for specific exam points, thereby improving their performance.

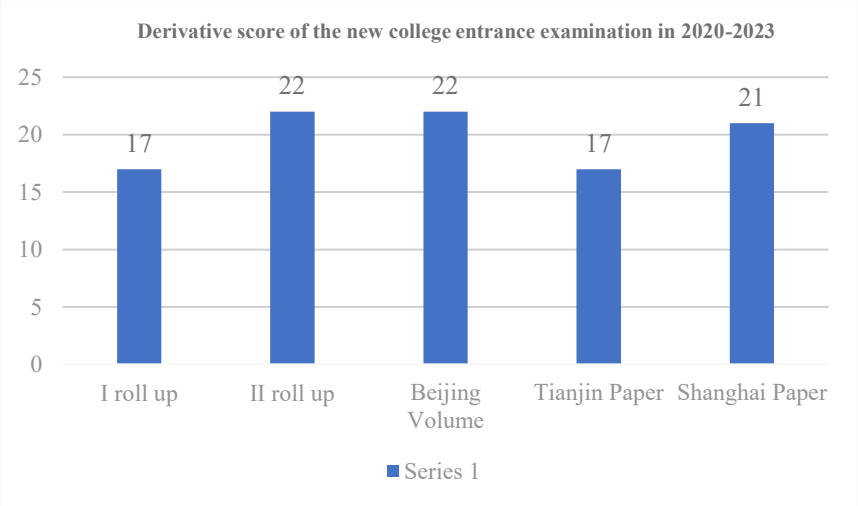


Figure 3: Percentage of Derivative in the New College Entrance Examination from 2020 to 2023

Data from the 2020-2023 National College Entrance Examination (Gaokao) reveals that multiple-choice questions carry 0 or 5 points, while fill-in-the-blank questions also have 0 or 5 points. Subjective questions consistently account for 12 points annually. As a result, derivative-related questions are a fixed component of the Gaokao, with their annual value remaining relatively stable at 17 or 22 points.

3.2 Statistics of Derivative Question Types and Exam Points in the New Gaokao

The new college entrance examination mathematics derivative knowledge points involve the function monotony, tangent equation, comparison, inequality constant, parameter value range, extreme value point shift problem.

Table 2: Distribution of Derivative Question Types in the National College Entrance Examination (Gaokao) Paper I from 2020 to 2023错误!使用“开始”选项卡将 二级标题 应用于要在此处显示的文字。

a particular year	choice question	gap filling	subjective item	total value
2020	monotonicity	tangential equation	monotonicity	22
2021	Compare	Parameter range	inequality holds	22
2022	Compare	tangential equation	extreme point shift	22
2023	Parameter range	monotonicity	monotonicity	22

Table 3: Distribution of Derivative Question Types in the National College Entrance Examination (Gaokao) Paper II from 2020 to 2023错误!使用“开始”选项卡将 二级标题 应用于要在此处显示的文字。

a particular year	choice question	gap filling	subjective item	total value
2020	monotonicity	tangential equation	monotonicity	22
2021	monotonicity	Parameter range	The inequality is always true.	22
2022	Compare	tangential equation	extreme point shift	22
2023	The inequality is always true.	/	extreme point shift	17

Table 4: Distribution of Derivative Question Types in the Beijing Version of the New College Entrance Examination from 2020 to 2023错误!使用“开始”选项卡将 二级标题 应用于要在此处显示的文字。

a particular year	choice question	gap filling	subjective item	total value
2020	Compare	/	monotonicity	20
2021	Compare	tangential equation	inequality holds	24
2022	monotonicity	Parameter range	monotonicity	24
2023	/	/	Parameter range	15

Table 5: Distribution of Derivative Question Types in Tianjin Papers of the New College Entrance Examination from 2020 to 2023错误!使用“开始”选项卡将 二级标题 应用于要在此处显示的文字。

a particular year	choice question	gap filling	subjective item	total value
2020	/	/	inequality holds	15
2021	/	/	monotonicity	15
2022	Compare	/	extreme point shift	20
2023	tangential equation	/	inequality holds	20

Table 6: Distribution of Derivative Question Types in the Shanghai Version of the New College Entrance Examination from 2020 to 2023错误!使用“开始”选项卡将 二级标题 应用于要在此处显示的文字。

a particular year	choice question	gap filling	subjective item	total value
2020	/	/	extreme point shift	15
2021	Compare	tangential equation	Parameter range	25
2022	/	Parameter range	monotonicity	20
2023	monotonicity	/	The inequality is always true.	20

Derivatives are a crucial concept in high school mathematics and a mandatory component of the National College Entrance Examination (Gaokao) mathematics section. To facilitate analysis of the score distribution across Gaokao topics, this paper categorizes derivative-related exam questions into four major types: investigating function monotonicity, solving extremum point shifts, proving inequality validity, and determining parameter ranges. Through detailed analysis of these four question types and statistical examination of test questions, the key assessment areas for derivatives are clearly identified. The specific statistics are as follows:

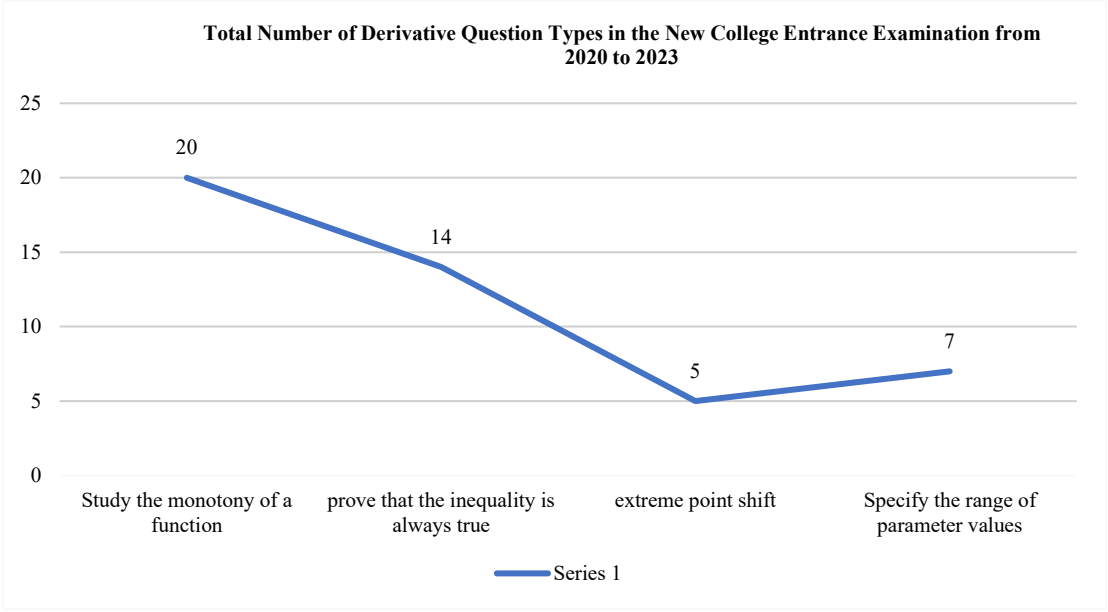


Figure 3: Line graph of the total number of questions on derivatives in the new college entrance examination from 2020 to 2023

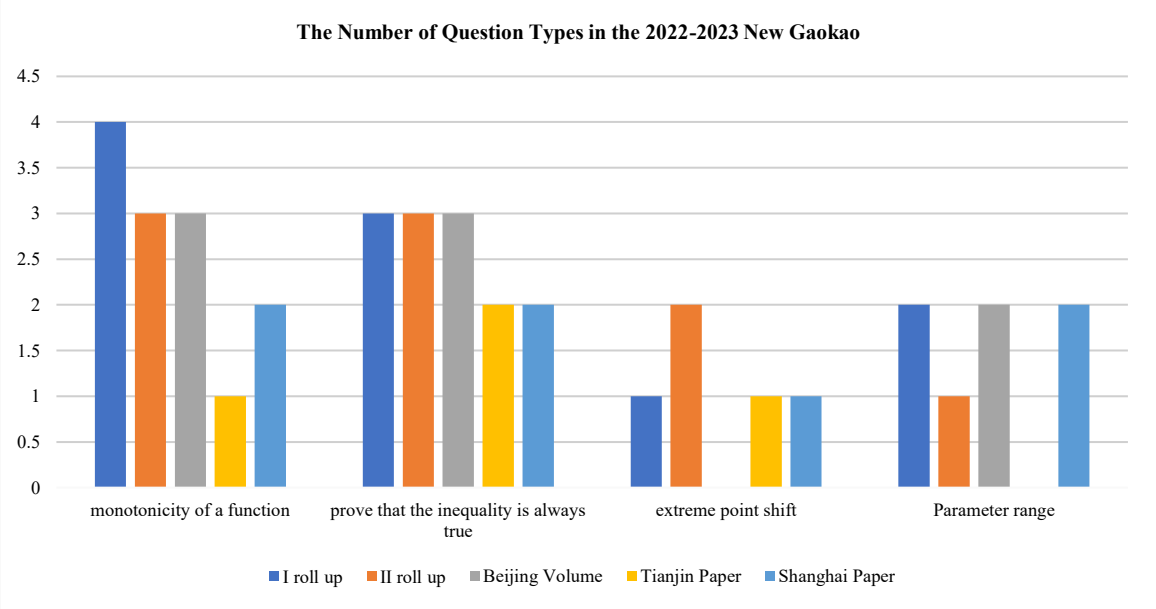


Figure 4: Statistics of the number of new Gaokao test types in 2022-2023

Statistical analysis reveals that in the 2022-2023 National College Entrance Examination (Gaokao), multiple-choice questions predominantly tested function monotonicity and demonstrated the constant validity of inequalities. Over the past five years, fill-in-the-blank questions have consistently focused on function monotonicity. Subjective questions generally consisted of two sub-questions, with a few comprising three, demonstrating clear examination points and stable question types. The first sub-question primarily assessed function monotonicity, while the second sub-question examined proving inequality validity, analyzing extremum shifts, and determining parameter ranges.

Table 7: Difficulty Analysis of Derivative Questions in the National College Entrance Examination Paper I from 2020 to 2023

a particular year	choice question	gap filling	subjective item	total value
2020	Simple questions	Simple questions	difficult problem	22
2021	Medium	Medium	difficult problem	22
2022	Medium	Simple questions	difficult problem	22
2023	difficult problem	Simple questions	difficult problem	22

Table 8: Difficulty Analysis of Derivative Questions in the National College Entrance Examination (Gaokao) Paper II (2020-2023)

a particular year	choice question	gap filling	subjective item	total value
2020	Medium	Medium	difficult problem	22
2021	Simple questions	difficult problem	difficult problem	22
2022	Medium	Simple questions	difficult problem	22
2023	difficult problem	/	difficult problem	17

Table 9: Difficulty Analysis of Derivative Questions in the Beijing Version of the New College Entrance Examination from 2020 to 2023

a particular year	choice question	gap filling	subjective item	total value
2020	Medium	/	difficult problem	20
2021	Simple questions	Simple questions	difficult problem	24
2022	Simple questions	Medium	difficult problem	24
2023	/	/	difficult problem	15

Table 10: Difficulty Analysis of Derivative Questions in Tianjin Papers of the New College Entrance Examination from 2020 to 2023

a particular year	choice question	gap filling	subjective item	total value
2020	/	/	difficult problem	15
2021	/	/	difficult problem	15
2022	Medium	/	difficult problem	20
2023	Simple questions	/	difficult problem	20

Table 11: Difficulty Analysis of Derivative Questions in Shanghais New College Entrance Examination from 2020 to 2023

a particular year	choice question	gap filling	subjective item	total value
2020	/	/	difficult problem	15
2021	Medium	Simple questions	difficult problem	25
2022	/	Medium	difficult problem	20
2023	Simple questions	/	difficult problem	20

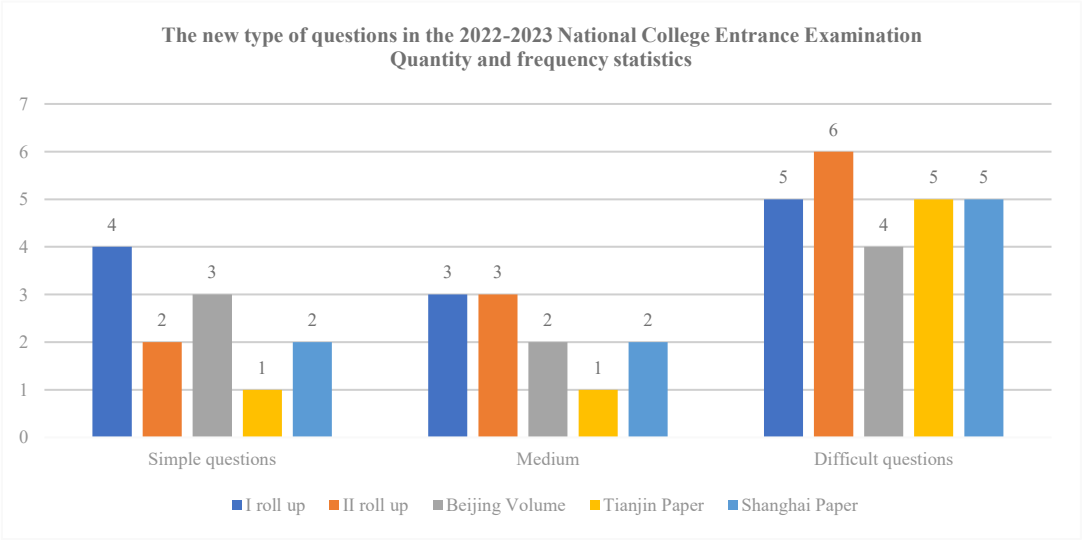


Figure 5: Statistics of the Number and Difficulty of Derivative Question Types in the New College Entrance Examination from 2022 to 2023

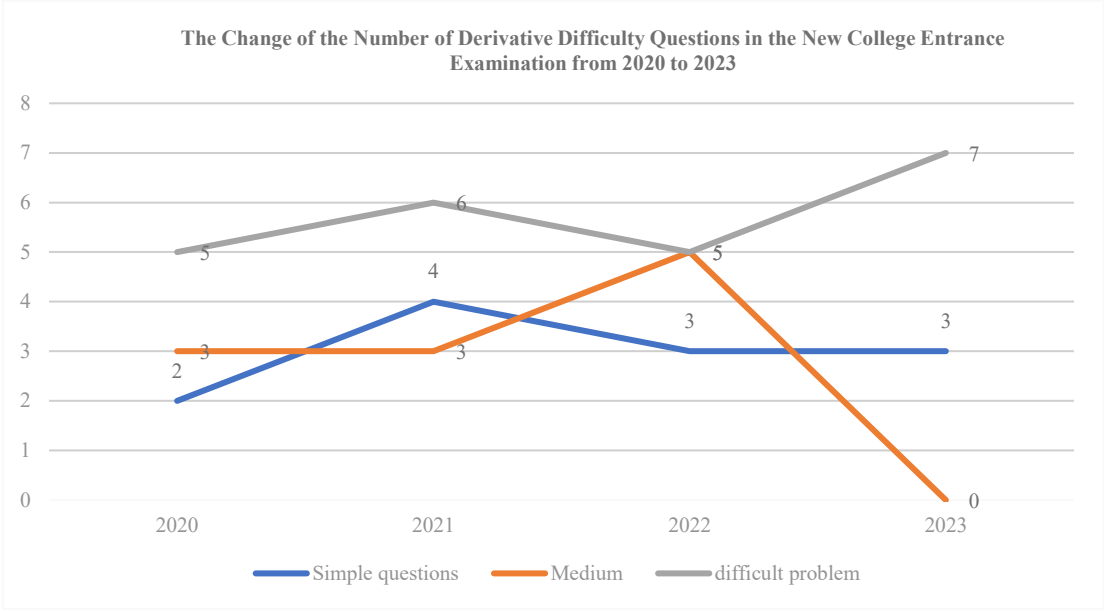


Figure 6: Line graph of the number of derivative difficulty questions in the new college entrance examination from 2020 to 2023

Based on the difficulty analysis of the question types, multiple-choice and fill-in-the-blank questions primarily consist of basic problems, while subjective questions remain the most challenging. The annual difficulty level remains relatively stable at a moderate level. As shown in Figure 2-8, the number of simple questions stays consistent, moderate questions show fluctuations, and difficult questions gradually increase. Notably, seven difficult questions appeared in the five exam papers of 2023. This indicates that the difficulty of derivatives in the college entrance exam has been rising annually. For high school students, this means they must master these concepts thoroughly, regularly practice derivative problems, and develop proficiency in various question types.

3.3 Analysis of the Comprehensive Difficulty of Derivative in the New College Entrance Examination

It is very necessary to analyze the composition and comprehensive difficulty of the test paper. Good questions can improve students logical reasoning ability. Therefore, in order to better grasp the knowledge points, we can quickly understand the meaning of the questions.

(1) The Model of the Comprehensive Difficulty of "Derivative"

The comprehensive difficulty model is a mathematical model for evaluating the complexity of a project or subject. It can help people better understand and quantify the difficulty of a task, and provide a basis for the task.But the five difficulty factors in Baos model are not deep enough. So in this paper, we use Wus model, which includes background factor, parameter level, operation level, reasoning ability, knowledge content, problem solving thinking mode and cognitive level.

(2) Comprehensive Difficulty Analysis of "Derivative"

In this statistical analysis, two sub-questions from the derivative problem section of each test paper are combined into a single question for evaluation. The following calculations are based on Wu Xiaopengs Comprehensive Difficulty Model. [21] The statistics are as follows:

Table 12: Comprehensive Statistics of Conic Curve Questions in the New Gaokao Papers from 2020 to 2023

factor	horizontal	Number of questions					percentage (%)					Comprehensive difficulty coefficient				
		I roll up	II roll up	Beijing Volume	Tianjin Paper	Shanghai Paper	I roll up	II roll up	north the capital of a country roll up	sky saliva roll up	falling-rising tone Sea roll up	I roll up	II roll up	Beijing Volume	Tianjin Paper	Shanghai Paper
background factor	No background	13	11	9	6	7	100.0	100.0	100.0	100.0	87.50	1	1	1	1	1.28
	Life background	0	0	0	0	1	0.00	0.00	0.00	0.00	12.50					
	Scientific background	0	0	0	0	0	0.00	0.00	0.00	0.00	0.00					
	Integrated Background	0	0	0	0	0	0.00	0.00	0.00	0.00	0.00					
parameter horizontal	No parameters	5	3	4	0	1	38.46	27.27	44.44	0.00	12.50	2	2.18	1.78	2.83	2.38
	Simple parameters	3	3	3	1	3	23.08	27.27	33.33	16.66	37.50					
	Complex parameters	5	5	2	5	4	38.46	45.46	22.23	83.34	50.00					
operation horizontal	Simple number	1	0	2	0	1	7.69	0.00	22.22	0.00	12.50	1.77	3.36	2.56	3.17	3
	complex number	2	1	2	1	0	15.38	9.10	22.22	16.66	0.00					
	Simple Symbol	2	5	3	3	5	15.38	45.45	33.34	50.00	62.50					
	complex symbol	8	5	2	2	2	61.55	45.45	22.22	33.34	25.00					
inference ability	Simple Reasoning	3	2	1	0	1	23.07	18.18	11.11	0.00	12.50	2.30	2.27	2.44	2.83	2.38
	general reasoning	3	4	3	1	3	23.07	36.36	33.33	16.67	37.50					
	complex reasoning	7	5	5	5	4	53.86	45.46	55.56	83.33	50.00					
knowledge content	A few knowledge points	3	2	2	0	1	23.07	18.18	22.22	0.00	12.50	2.30	2.09	2.22	2.5	2.38
	Intermediate knowledge points	3	6	3	3	3	23.07	54.54	33.33	50.00	37.50					
	A wealth of knowledge	7	3	4	3	4	53.86	27.28	44.45	50.00	50.00					
Problem-solving thinking patterns	forward thinking	7	7	5	4	5	53.84	63.63	55.55	66.67	62.50	1.46	1.36	1.44	1.33	1.36
	reversed thinking	6	4	4	2	3	46.16	36.37	44.45	33.33	37.50					
knowledge horizontal	memorize	5	5	2	1	1	38.46	45.45	22.22	16.67	12.50	1.77	1.73	2.22	2.17	2.25
	utilize	6	4	3	3	4	46.15	36.36	33.33	50.00	50.00					
	aggregate analysis	2	2	4	2	3	15.39	18.19	44.45	33.33	37.50					
condition content	Single condition	4	1	4	2	2	30.77	9.09	44.44	33.33	25.00	1.85	2.09	1.67	2	2
	Two conditions	7	8	4	2	4	53.85	72.72	44.44	33.33	50.00					
	Three or more	2	2	1	2	2	15.38	18.19	11.12	33.33	25.00					
character reading quantity	Fewer characters	6	5	2	1	3	46.15	45.45	27.22	16.67	37.50	1.77	1.91	2.11	2.17	2
	Medium	4	2	4	3	2	30.76	18.19	44.44	50.00	25.00					
	Large number of characters	3	4	3	2	3	23.09	36.36	28.34	33.33	37.50					

① Comparison of different levels of each factor

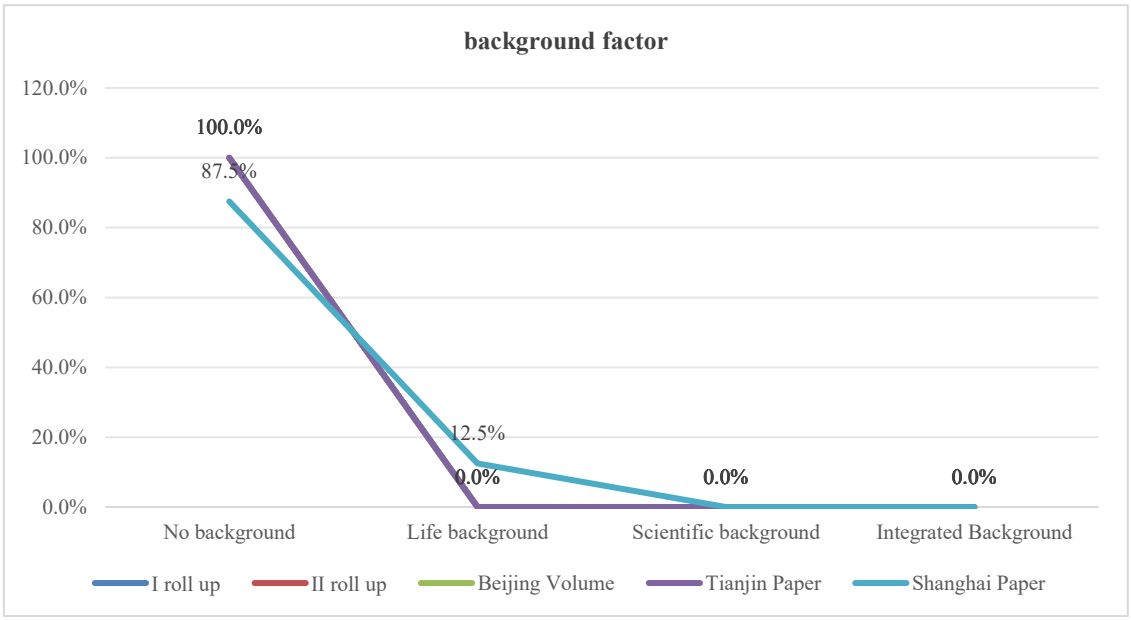


Figure 7: Line chart showing changes in the levels of background factors

1) The background factor line chart clearly shows that from 2020 to 2023, the derivative questions in the new college entrance exam papers generally did not involve scientific or comprehensive contexts, indicating weaker emphasis on this aspect. To improve test design, educators could appropriately integrate real-life scenarios into questions, helping students appreciate the practical applications of mathematics and fostering greater interest in the subject.

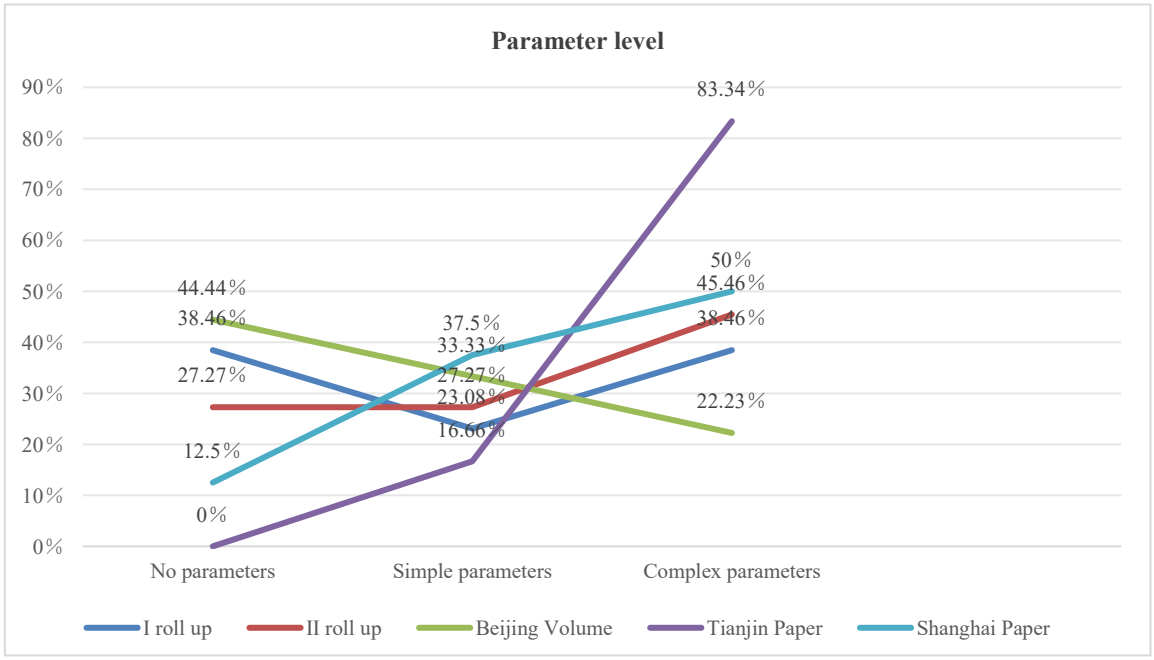


Figure 8: Line chart of parameter level changes

2) The parameter difficulty line chart reveals that from 2020 to 2023, the derivative questions in the new college entrance exam papers predominantly covered both simple and complex parameters, with a consistent trend. Thus, students must master these types of questions thoroughly, requiring them to develop proficiency in parameter setting and solving.

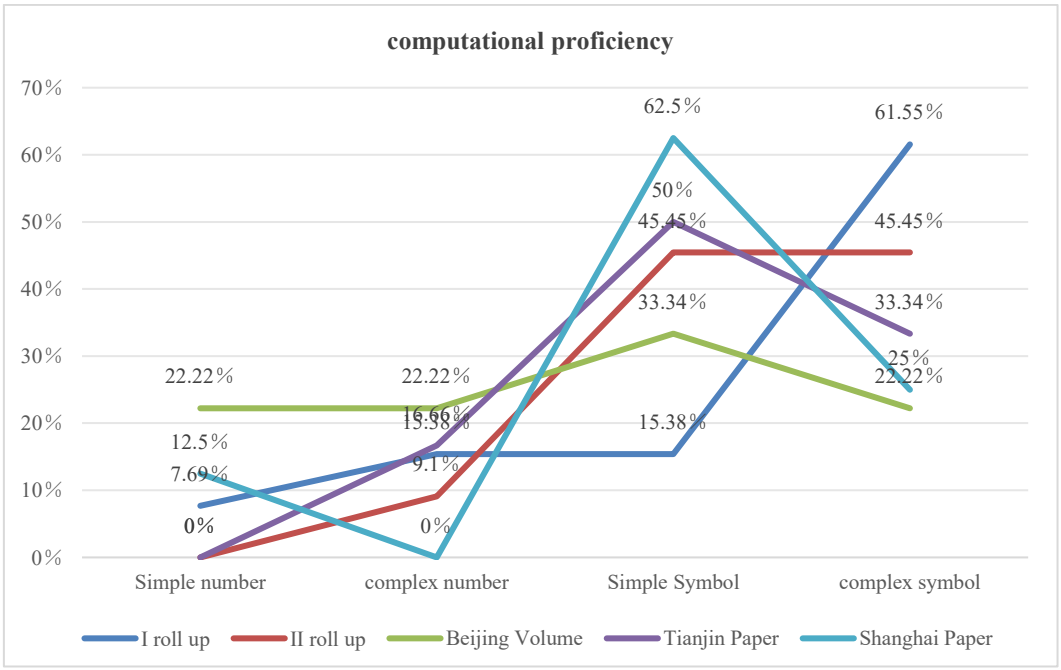


Figure 9: Line graph of changes in operational level

3) The line graph of computational difficulty clearly shows that the national exam primarily assesses computational skills through simple and complex symbols, mainly in subjective questions. Therefore, students must be meticulous when solving derivative problems, clearly outlining each step to avoid errors in calculations.

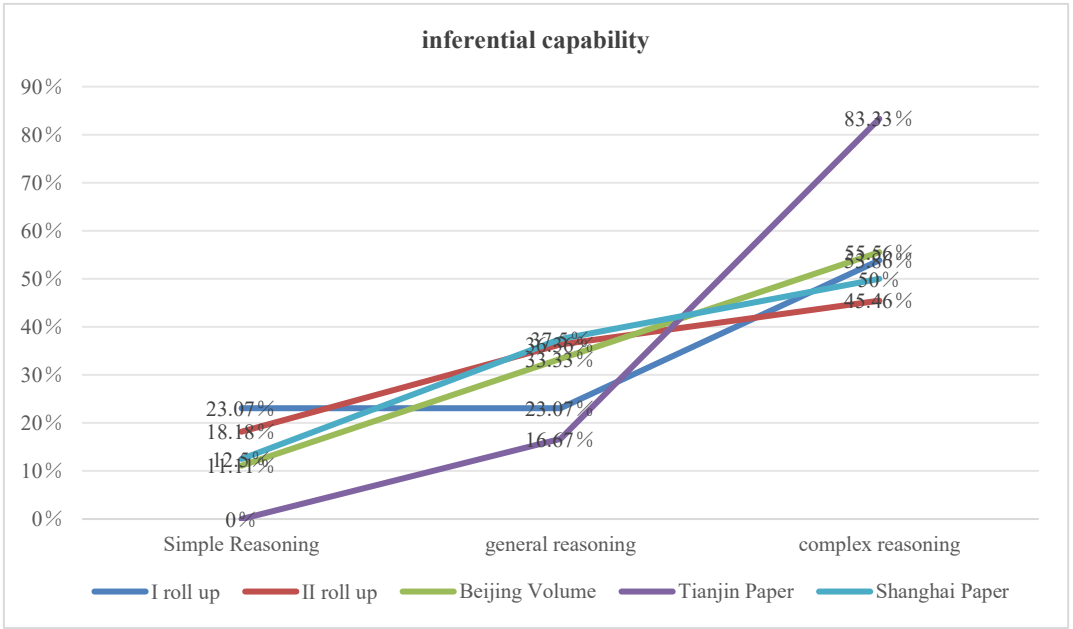


Figure 10: Change in Reasoning Ability (Line Chart)

4) The difficulty of reasoning is evident from the line graph, which shows that derivative problems involve complex reasoning processes, indicating their high difficulty level and rigorous testing of students logical reasoning abilities. Therefore, it is crucial to emphasize the cultivation of students logical reasoning skills. Particularly in the subjective questions that are always tested in derivative problems, students must have a solid foundation and a clear problem-solving approach to effectively demonstrate their reasoning process.

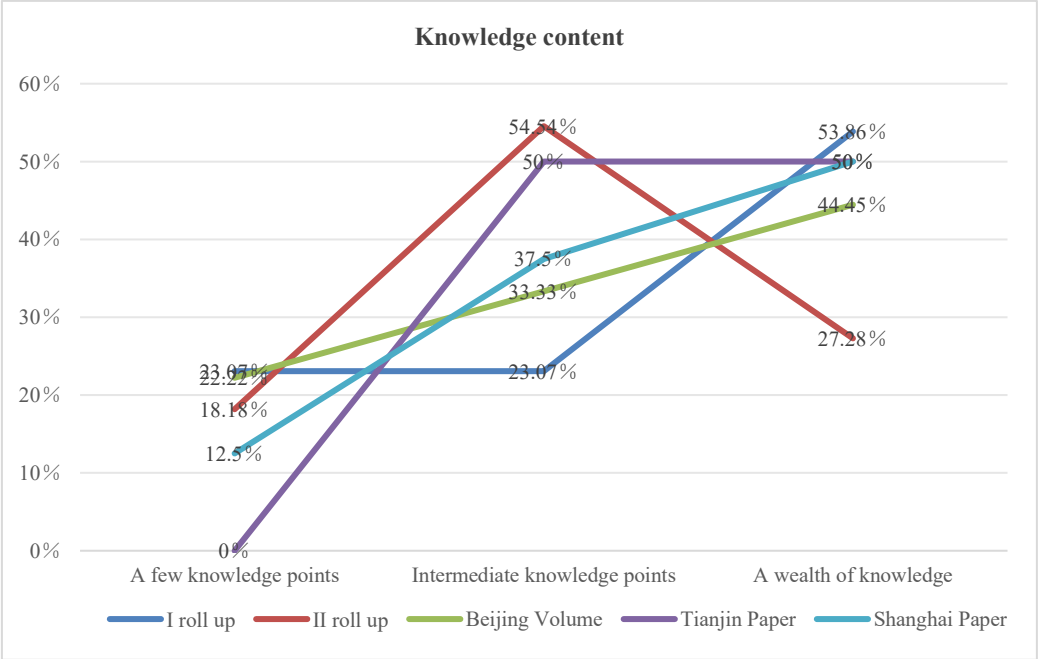


Figure 11: Knowledge Content Change Line Chart

5) The knowledge point line graph reveals that the derivative questions in the national exam cover a wide range of complex concepts, often involving multiple knowledge points. Students must analyze the given conditions and identify the relevant knowledge points to solve problems efficiently.

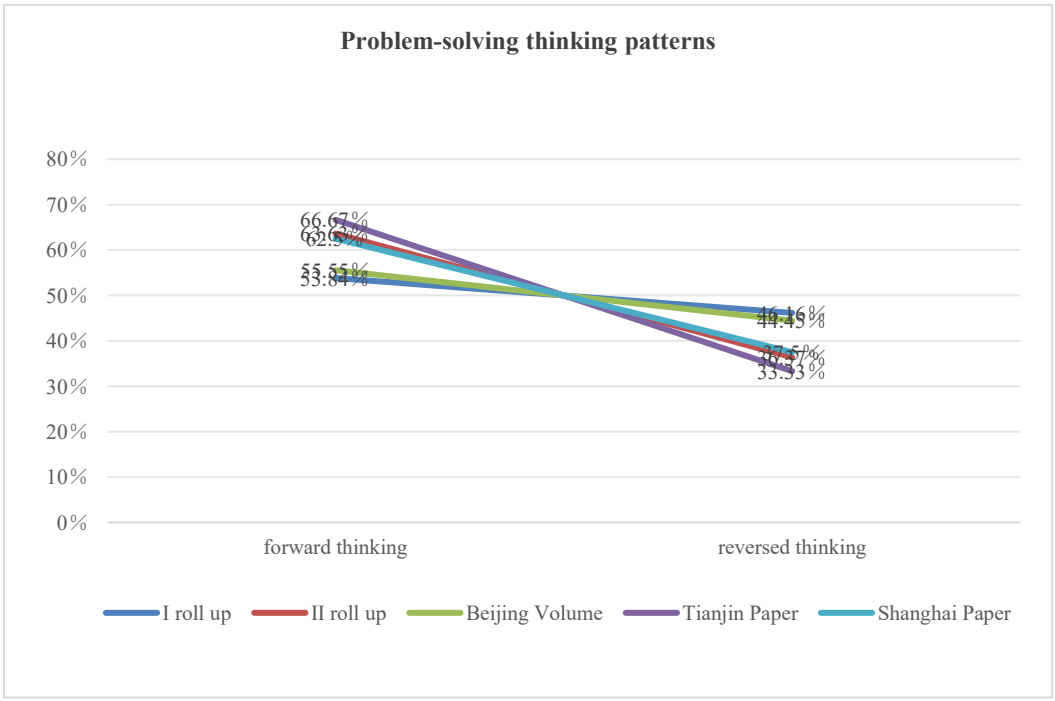


Figure 12: Changes in Problem-Solving Thinking Patterns (Line Chart)

6) The line chart of thinking patterns clearly shows that the test questions predominantly assess linear thinking, with fewer questions evaluating divergent thinking. Consequently, derivative problems require students to derive solutions from given conditions through systematic reasoning. This approach involves progressively exploring problem-solving methods while maintaining established knowledge frameworks and logical progression.

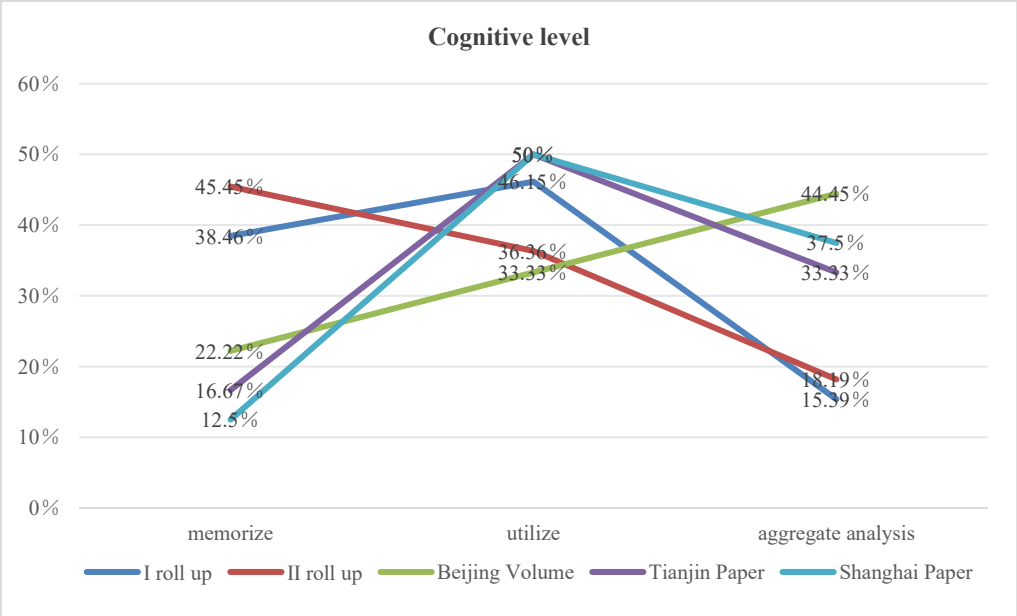


Figure 13: Line graph of cognitive level changes

7) The cognitive level line chart reveals that the national exam questions demonstrate a relatively stable comprehensive level, requiring students to apply knowledge flexibly and demonstrate strong analytical skills. The questions are challenging, demanding thorough consideration from examinees.

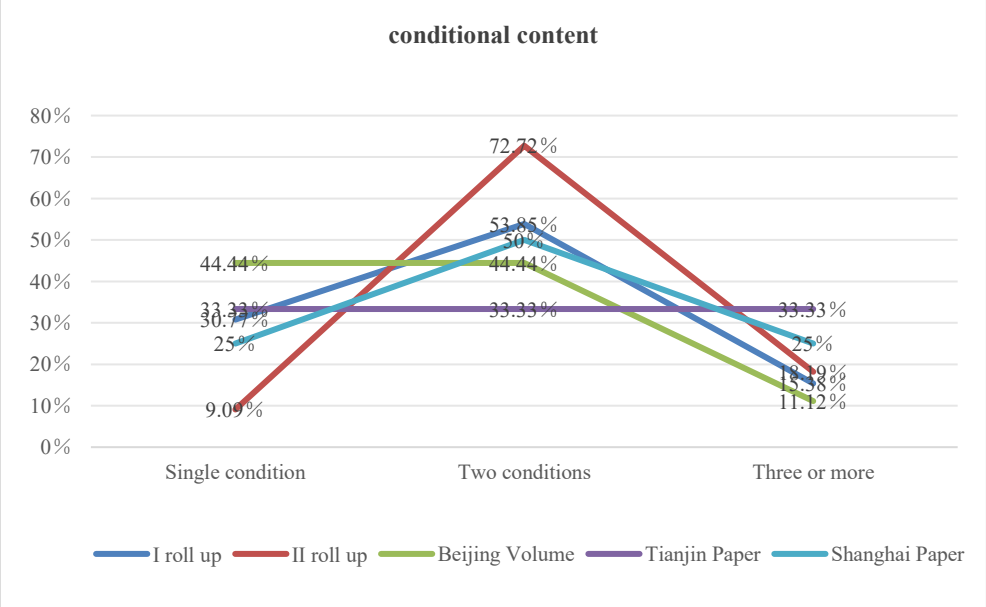


Figure 14: Line graph of conditional content variation

8) The conditional quantity line chart indicates that major questions typically contain 2 to 3 conditions. Questions with multiple conditions are generally simpler, focusing solely on computational skills, allowing students to identify key points and solve them effectively. For questions with fewer conditions, students need to think carefully and deliberate repeatedly, ensuring all conditions are utilized to guarantee accuracy.

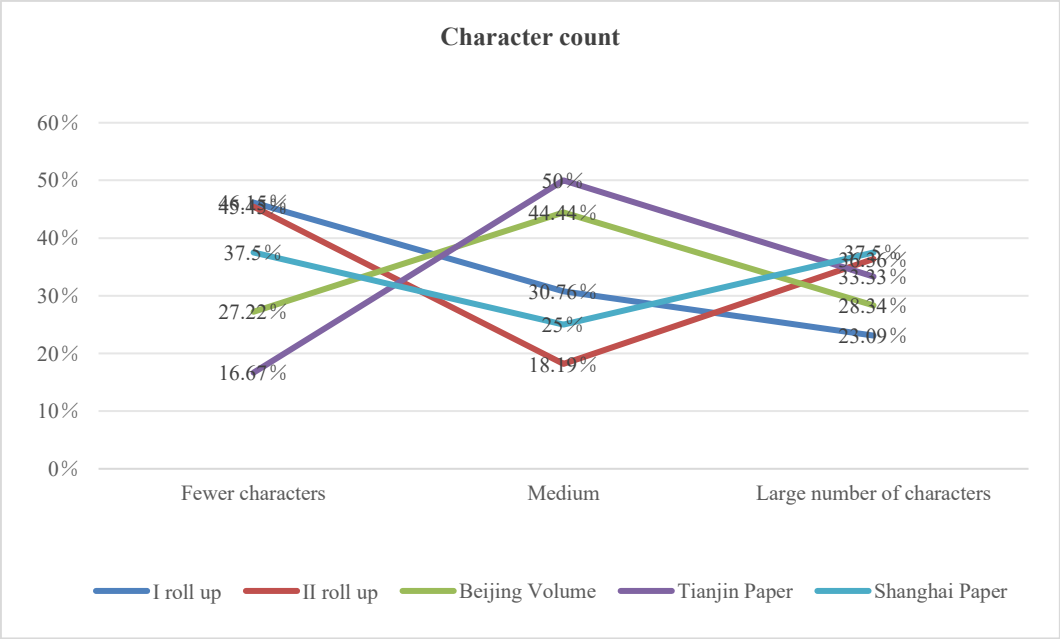


Figure 15: Comparative line chart of character recognition rates

9) The bar chart of character count shows that test questions vary in length, with most questions containing fewer characters for quick reading. Students must first understand the exam points before identifying them to solve the problems. So students should think about the solution method and steps while reading.

② Comparison of overall difficulty levels

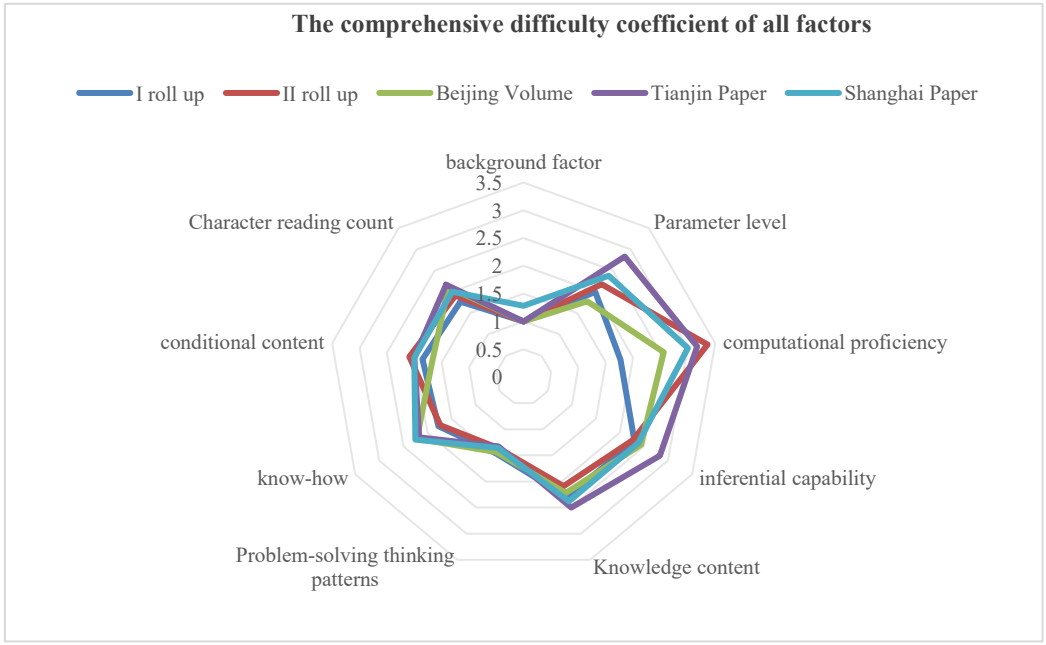


Figure 16: Radar Chart of Comprehensive Difficulty Coefficient of Derivative Questions in the New Gaokao from 2022 to 2023

The above radar chart analysis is as follows:

The 2022-2023 new college entrance examination derivative questions demonstrated comparable characteristics in background factors, parameter levels, conditional content, cognitive requirements, problem-solving approaches, computational proficiency, and character reading difficulty coefficients, with overall difficulty levels remaining consistent. Analysis reveals that these derivative questions primarily assess students computational skills, reasoning abilities, and knowledge mastery, thus demanding higher comprehensive competencies. This requires

students to possess strong logical reasoning capabilities and computational proficiency.

③ brief summary

As the analysis shows, derivatives in the National Mathematics Exam for the new college entrance system present complex concepts with consistent difficulty levels and scoring weightings, making them a major learning challenge. To master derivatives, students must develop computational skills and logical reasoning abilities. Computational proficiency is particularly crucial, as many students frequently make calculation errors in derivative problems, which can significantly impact their final scores. When substituting variables, students should exercise extreme caution—any miscalculation could prove counterproductive.

PROJECT FUND

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