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Least Squares Support Vector Machine Beamforming Algorithm

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Abstract: ***The article primarily proposes a beamforming method based on the Least Squares Support Vector Machine (LSSVM). Using a uniform horizontal line array as an example, it explores the design of plane wave beamforming under the far-field assumption. Unlike conventional methods of weighted steering vector and copied correlation beamforming for uniform horizontal line arrays, the article suggests using the Least Squares Support Vector Machine algorithm to establish a learning relationship between received and desired signals, replacing steering vectors with learned weights, and substituting copied correlation operations with kernel operations for beamforming. Simulation verification shows that compared to conventional beamforming methods, this approach achieves narrower mainlobe widths and lower sidelobe levels, providing theoretical support for further engineering applications.***

Keywords: Beamforming; Least Squares Method; Support Vector Machine; Far-field Plane Wave.

1. INTRODUCTION

In conventional beamforming problems [1], steering vectors are used with real signals for copy correlation. Steering vectors are typically divided into plane wave models based on near-field and far-field assumptions. However, in reality, the actual positions of array elements can shift, leading to incorrect construction of steering vectors and performance degradation of beamformers due to array mismatches [2-3]. Addressing these issues, Chen Pei et al. [4] introduced a criterion of progressive minimum variance, aimed at achieving sparse reconstruction of the interference-plus-noise covariance matrix and obtaining steering vector estimates in the desired direction, thereby determining the optimal steering vector for the beamformer. Cui Lin et al. [5] proposed a robust beamforming method using adaptive Gaussian-Legendre integration for covariance matrix reconstruction, aimed at determining the optimal steering vector. Martinez-Ramon was the first to propose applying Support Vector Machine (SVM) optimization techniques for estimating unknown parameters [6-7]. Building on Vapnik's SVM regression algorithm [8], this paper proposes a Least Squares Support Vector Machine beamforming method that establishes a learning relationship between the received and expected signals, using learned weights to replace the steering vectors, and kernel operations to replace copy correlation operations.

The desired signal can be constructed from the spectrum, either as a single frequency signal or a superposition of single frequency signals. The steering vectors are learned from actual data, rather than being predetermined by a model. It has good adaptive capabilities. The choice of the Least Squares Support Vector Machine is due to its strong mathematical logic, few adjustable parameters, high interpretability, and fast computational speed.

2. METHODOLOGY

2.1 Research Approach

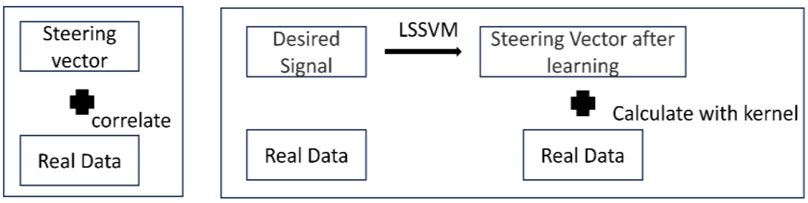


Figure 1: Research approach flowchart of this paper

As shown in Figure 1, this study transitions from classical copy correlation to regression processing of real data. The advantage of choosing the Least Squares Support Vector Machine lies in its ability to transform the problem into a constrained optimization problem, enhancing the model's interpretability, while least squares can increase the solution speed. Below, we start with the derivation from the Least Squares Support Vector Machine to obtain the learned steering vectors.

The output of a beamformer can be expressed as a linear combination of the outputs from each array element.

$$d[n] = w^H x[n] + e[n] \quad (1)$$

$d[n]$ is the expected signal output after being processed by the beamformer, w represents the weighted vector of each array element, $x[n]$ is the signal received by each array element, $e[n]$ is the output error of the signal. Among these, the received signal $x[n]$ includes the desired signal $s[n]$ and noise $v[n]$.

$$x[n] = s[n] + v[n] \quad (2)$$

The desired observable signal is $s[n]$, but the actual measurable data is $x[n]$.

2.2 Least Squares Support Vector Machine Beamforming

For linear beamforming problems, the design function for linear beamformers is as follows:

$$f(x) = w^H x + b \quad (3)$$

This paper establishes a nonlinear learning relationship between the desired signal $d[n]$ and the array received signal $x[n]$, introducing a mapping function $\varphi(x): R \to H$, where R represents the input data space of x, and H represents the feature data space of $\varphi(x)$. Therefore, the LSSVM regression problem can be expressed as:

$$\begin{aligned} &\min J(w,\xi) = \frac{1}{2}\|w\|^2 + \frac{1}{2}\gamma \sum_{i=1}^{l} \xi_i^2 \\ &s.t. \quad f(x) = w^H \varphi(x_i) + b + \xi_i \quad i = 1,2,\dots,l \end{aligned} \quad (4)$$

Where ξ_i is the slack variable, γ is the regularization parameter that balances fitting error and model complexity. l is the number of data points under a single snapshot.

For the problem described above, a Lagrangian function is introduced, resulting in the following equation:

$$L(w,b,\xi_i,\alpha) = J - \sum_{i=1}^{l} \alpha_i [w^H \varphi(x_i) + b + \xi_i - y_i] \quad (5)$$

In the formula, α_i represents the Lagrange multipliers. Based on the optimization conditions, taking partial derivatives with respect to w, b, ξ_i, α_i results in:

$$\begin{aligned} &\frac{\partial \iota}{\partial w} = 0 \Rightarrow w = \sum_{i=1}^{l} \alpha_i \varphi(x_i) \\ &\frac{\partial \iota}{\partial b} = 0 \Rightarrow \sum_{i=1}^{l} \alpha_i = 0 \\ &\frac{\partial \iota}{\partial \xi_i} = 0 \Rightarrow \alpha_i = C\xi_i \quad i = 1,\dots,\iota \\ &\frac{\partial \iota}{\partial \alpha_i} = 0 \Rightarrow w^H \varphi(x_i) + b + \xi_i - y_i = 0 \quad i = 1,\dots,\iota \end{aligned} \quad (6)$$

To find the optimal values of α and b, the KKT conditions yield:

$$\begin{bmatrix} 0 & I' \\ I & K + \gamma^{-1} I \end{bmatrix} \begin{bmatrix} b \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 \\ y \end{bmatrix} \quad (7)$$

In the above equation, $y = [y_1, \dots, y_l]^H$; $\alpha = [\alpha_1, \dots, \alpha_l]^H$; $I = [I_1, \dots, I_l]^H$; K is the kernel matrice, where H denotes the conjugate transpose symbol. The element at the i-th row and j-th column is $K_{ij} = \langle \varphi(x_i) \cdot \varphi(x_j) \rangle, i,j = 1,\dots,\iota$. Finally, substituting α and b, which are solved from the linear system, into the least squares regression function is:

$$f(x) = w^H \varphi(x) + b = \sum_{i=1}^{l} \alpha_i K(x_i, x) + b \quad (8)$$

By introducing kernel function theory, it transforms the problem into high-dimensional feature space, turning it into a linear problem solution in high-dimensional space. The optimization problem is mapped to high-dimensional feature space for discussion; according to VC dimension theory, the high-dimensional feature space may be infinite-dimensional. Therefore, an explicit representation of the nonlinear mapping $\varphi()$is difficult, thus posing practical computational challenges. Kernel functions provide a feasible means for computations in high-dimensional spaces, involving only inner product operations for data points in the input space, introducing kernel functions that satisfy Mercer's condition allows the results of inner product operations in high-dimensional spaces to be directly defined in low-dimensional spaces.

$$K_{ij} = (x_i, x_j) = \langle \varphi(x_i) \cdot \varphi(x_j) \rangle \tag{9}$$

Because the Least Squares Support Vector Machine transforms the complex solution of the convex quadratic programming problem of the classic Support Vector Machine into the solution of a system of linear equations, this greatly simplifies the computational complexity and increases the speed of solution.

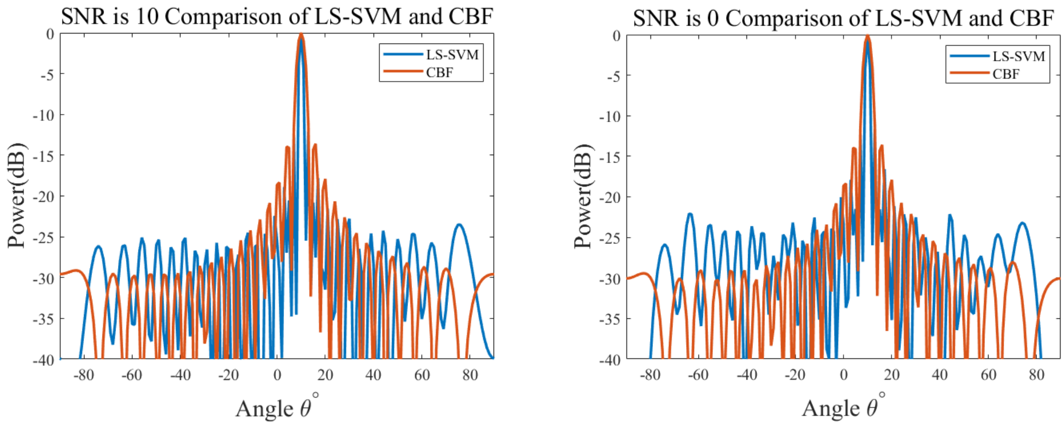
3. RESULTS AND DISCUSSION

3.1 Simulation Analysis of LSSVM Beamforming Under Different Signal-to-noise Ratios with Array Position Mismatch

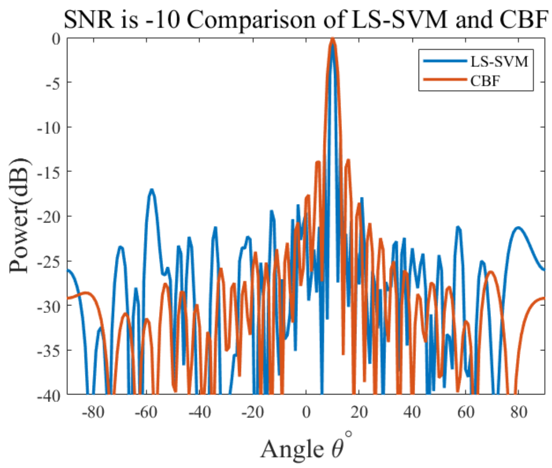
Compared to conventional beamforming, the advantages of LSSVM beamforming include narrower mainlobe widths and lower sidelobe levels, providing theoretical support for further engineering applications. Below, a comparative analysis is conducted of the LSSVM beamforming performance versus Conventional Beamforming (CBF) under far-field conditions with a uniform horizontal line array.

The simulation environment consists of a 30-element horizontal line array in a far-field setting, with a sound speed of 1500 m/s, a sampling frequency of 10 kHz, a half-wavelength array, a 1 kHz single-frequency sound source, and Gaussian additive noise. The angle of arrival is 10 degrees. At signal-to-noise ratios of 10, 0, and -10 dB, LSSVM uses a Gaussian kernel. Here, the effects of LSSVM beamforming are compared with those of conventional beamforming.

The results of the beamforming are shown in Figure 2 below.



(a) (b)



(c)

Figure 2: Comparison of LSSVM beamforming effects with conventional beamforming at different signal-to-noise ratios

The statistics for the mainlobe width and the first sidelobe level are shown in Table 1 below.

Table 1: Experimental data for LSSVM beamforming versus conventional beamforming at different signal-to-noise ratios

SNR	Mainlobe width	The first sidelobe level
10dB(CBF)	4.5°	-13.59 dB
10dB(LSSVM)	2°	-20.54dB
0dB(CBF)	4.5°	-13.59dB
0dB(LSSVM)	2°	-20.49dB
-10dB(CBF)	4.5°	-13.06dB
-10dB(LSSVM)	2°	-21.19dB

From the above statistical results, it is evident that LSSVM beamforming has a narrower mainlobe width and lower first sidelobe level compared to Conventional beamforming.

3.2 Simulation Analysis of SVM Beamforming with Different Kernel Functions

Next, we analyze the impact of different kernel functions on the beam patterns of the learned steering vectors. Figure 3 below shows the effects of LSSVM beam patterns with different kernel functions at the same signal-to-noise ratio.

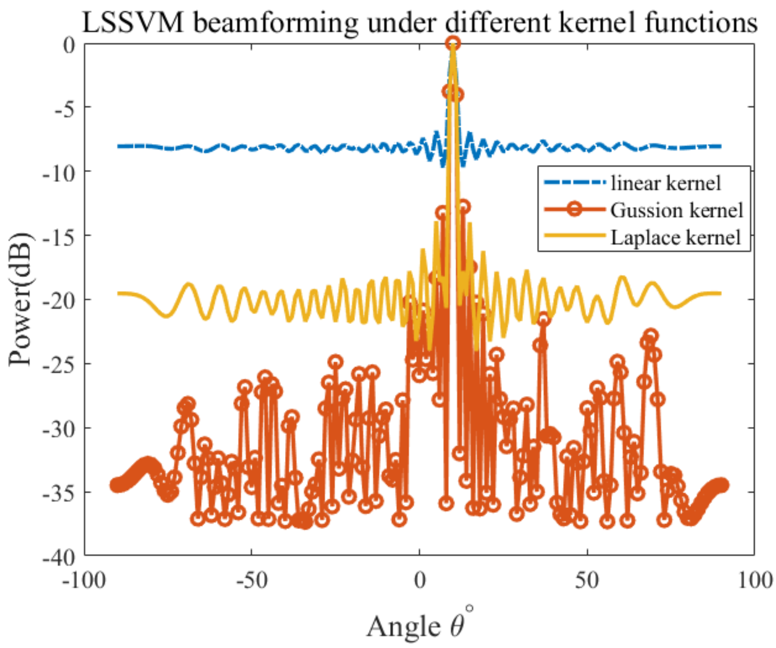


Figure 3: Effects of steering vector beam patterns under different types of kernel functions at the same signal-to-noise ratio

Under linear, Gaussian, and Laplacian kernels, the first sidelobe levels of beamforming are -6.87, -12.96, and -13.75 dB, respectively. It is evident that different kernel functions have a significant impact on LSSVM learning. The proper learning and selection of kernel functions affect the quality of the beamformer's performance. Designing appropriate kernel functions is an important direction for future development.

4. CONCLUSION

This paper explains the principles of the LSSVM algorithm, derives the design process of the LSSVM beamformer, and highlights the advantages of the LSSVM algorithm compared to traditional beamforming algorithms. Simulations were conducted to compare LSSVM beamforming with conventional beamforming at different signal-to-noise ratios, and the differences in beam patterns of LSSVM beamforming with different kernel functions were analyzed. The feasibility of applying the LSSVM method to beamforming techniques was demonstrated. This provides theoretical support for further engineering applications.

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