World Journal of Innovation and Modern Technology, Vol.6 Issue 4 (Aug) 2023
ISSN 2682-5910
10.53469/wjimt.2023.06(04).09

Proof of Riemann Paper (1859)

Vedatroyee Ghosh, Akash Sharma, Viraj Bansal

University of Engineering and Management, Gurukul, Sikar Road near Udaipuria Mod, Jaipur, Rajasthan 303807, India

Abstract: ***In 1859 Riemann defined the zeta function $\zeta(s)$. From Gamma function he derived the zeta function with Gamma function $\overline{\zeta}(s)$. $\overline{\zeta}(s)$ and $\zeta(s)$ are the two different functions. It is false that $\overline{\zeta}(s)$ replaces $\zeta(s)$. After him later mathematicians put forward Riemann hypothesis(RH) which is false. The Jiang function $J_n(\omega)$ can replace RH.***

Keywords: Riemann; zeta function.

1. INTRODUCTION

AMS mathematics subject classification: Primary 11M26.

In 1859 Riemann defined the Riemann zeta function (RZF)[1]

$$\zeta(s)=\prod_P(1-P^{-s})^{-1}=\sum_{n=1}^{\infty}\frac{1}{n^s}\ , \tag{1}$$

where $s=\sigma+ti, i=\sqrt{-1}$, σ and t are real, P ranges over all primes. RZF is the function of the complex variable s in $\sigma\ge 0, t\ne 0$, which is absolutely convergent.

In 1896 J. Hadamard and de la Vallee Poussin proved independently [2]

$$\zeta(1+ti)\ne 0\,. \tag{2}$$

In 1998 Jiang proved [3]

$$\zeta(s)\ne 0\,, \tag{3}$$

where $0\le\sigma\le 1$.

Riemann paper (1859) is false [1] We define Gamma function [1, 2]

$$\Gamma\left(\frac{s}{2}\right)=\int_0^{\infty} e^{-t}t^{\frac{s}{2}-1}dt\,. \tag{4}$$

For $\sigma>0$. On setting $t=n^2\pi x$, we observe that

$$\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)n^{-s}=\int_0^{\infty} x^{\frac{s}{2}-1}e^{-n^2\pi x}dx\,. \tag{5}$$

Hence, with some care on exchanging summation and integration, for $\sigma>1$,

$$\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)\overline{\zeta}(s)=\int_0^{\infty} x^{\frac{s}{2}-1}\left(\sum_{n=1}^{\infty}e^{-n^2\pi x}\right)dx=\int_0^{\infty} x^{\frac{s}{2}-1}\left(\frac{\vartheta(x)-1}{2}\right)dx \tag{6}$$

where $\overline{\zeta}(s)$ is called Riemann zeta function with gamma function rather than $\zeta(s)$,

$$\vartheta(x):=\sum_{n=-\infty}^{\infty}e^{-n^2\pi x}\,, \tag{7}$$

is the Jacobi theta function. The functional equation for $\vartheta(x)$ is

$$x^{\frac{1}{2}}\vartheta(x)=\vartheta(x^{-1}), \tag{8}$$

and is valid for $x>0$.

Finally, using the functional equation of $\vartheta(x)$, we obtain

$$\overline{\zeta}(s)=\frac{\pi^{\frac{s}{2}}}{\Gamma\left(\frac{s}{2}\right)}\left\{\frac{1}{s(s-1)}+\int_{1}^{\infty}(x^{\frac{s}{2}-1}+x^{-\frac{s}{2}-\frac{1}{2}})\cdot(\frac{\vartheta(x)-1}{2})dx\right\}. \tag{9}$$

From (9) we obtain the functional equation

$$\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)\overline{\zeta}(s)=\pi^{-\frac{1-s}{2}}\Gamma\left(\frac{1-s}{2}\right)\overline{\zeta}(1-s)\,. \tag{10}$$

The function $\overline{\zeta}(s)$ satisfies the following

1) $\overline{\zeta}(s)$ has no zero for $\sigma>1$;

2) The only pole of $\overline{\zeta}(s)$ is at $s=1$; it has residue 1 and is simple;

3) $\overline{\zeta}(s)$ has trivial zeros at $s=-2,-4,...$ but $\zeta(s)$ has no zeros;

4) The nontrivial zeros lie inside the region $0\le\sigma\le1$ and are symmetric about both the vertical line $\sigma=1/2$.

The strip $0\le\sigma\le1$ is called the critical strip and the vertical line $\sigma=1/2$ is called the critical line.

Conjecture (The Riemann Hypothesis). All nontrivial zeros of $\overline{\zeta}(s)$ lie on the critical line $\sigma=1/2$, which is false. [3]

$\overline{\zeta}(s)$ and $\zeta(s)$ are the two different functions. It is false that $\overline{\zeta}(s)$ replaces $\zeta(s)$, Pati proved that is not all complex zeros of $\overline{\zeta}(s)$ lie on the critical line: $\sigma=1/2$ [4].

Schadeck pointed out that the falsity of RH implies the falsity of RH for finite fields [5, 6]. RH is not directly related to prime theory. Using RH mathematicians prove many prime theorems which is false. In 1994 Jiang discovered Jiang function $J_n(\omega)$ which can replace RH, Riemann zeta function and L-function in view of its proved feature: if $J_n(\omega)\ne0$ then the prime equation has infinitely many prime solutions; and if $J_n(\omega)=0$, then the prime equation has finitely many prime solutions. By using $J_n(\omega)$ Jiang proves about 600 prime theorems including the Goldbach's theorem, twin prime theorem and theorem on arithmetic progressions in primes[7,8].

In the same way we have a general formula involving $\overline{\zeta}(s)$

$$\int_0^{\infty}x^{s-1}\sum_{n=1}^{\infty}F(nx)dx=\sum_{n=1}^{\infty}\int_0^{\infty}x^{s-1}F(nx)dx=\sum_{n=1}^{\infty}\frac{1}{n^s}\int_0^{\infty}y^{s-1}F(y)dy=\overline{\zeta}(s)\int_0^{\infty}y^{s-1}F(y)dy\,, \tag{11}$$

where $F(y)$ is arbitrary.

From (11) we obtain many zeta functions $\overline{\zeta}(s)$ which are not directly related to the number theory.

The prime distributions are order rather than random. The arithmetic progressions in primes are not directly related to ergodic theory ,harmonic analysis, discrete geometry, and combinatories. Using the ergodic theory Green and Tao prove that there exist infinitely many arithmetic progressions of length k consisting only of primes which is false [9, 10, 11]. Fermat's last theorem (FLT) is not directly related to elliptic curves. In 1994 using elliptic curves Wiles proved FLT which is false [12]. There are Pythagorean theorem and FLT in the complex hyperbolic functions and complex trigonometric functions. In 1991 without using any number theory Jiang proved FLT which is Fermat's marvelous proof[7, 13].

Primes Represented by $P_1^n+mP_2^n$ [14]

(1) Let $n=3$ and $m=2$. We have

$$P_3=P_1^3+2P_2^3\,.$$

We have Jiang function

$$J_3(\omega)=\prod_{3\le P}(P^2-3P+3-\chi(P))\neq 0,$$

Where $\chi(P)=2P-1$ if $2^{\frac{P-1}{3}}\equiv 1 \pmod P$; $\chi(P)=-P+2$ if $2^{\frac{P-1}{3}}\not\equiv 1 \pmod P$; $\chi(P)=1$ otherwise.

Since $J_n(\omega)\neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.
We have the best asymptotic formula

$$\pi_2(N,3)=\left|\{P_1,P_2:P_1,P_2\le N,P_1^3+2P_2^3=P_3\text{ prime}\}\right|\sim\frac{J_3(\omega)\omega}{6\Phi^3(\omega)}\frac{N^2}{\log^3 N}=\frac{1}{3}\prod_{3\le P}\frac{P(P^2-3P+3-\chi(P))}{(P-1)^3}\frac{N^2}{\log^3 N}.$$

where $\omega=\prod_{2\le P}P$ is called primorial, $\Phi(\omega)=\prod_{2\le P}(P-1)$.

It is the simplest theorem which is called the Heath-Brown problem [15].

(2) Let $n=P_0$ be an odd prime, $2|m$ and $m\neq\pm b^{P_0}$.

we have

$$P_3=P_1^{P_0}+mP_2^{P_0}$$

We have

$$J_3(\omega)=\prod_{3\le P}(P^2-3P+3-\chi(P))\neq 0,$$

where $\chi(P)=-P+2$ if $P|m$; $\chi(P)=(P_0-1)P-P_0+2$ if $m^{\frac{P-1}{P_0}}\equiv 1 \pmod P$;

$\chi(P)=-P+2$ if $m^{\frac{P-1}{P_0}}\not\equiv 1 \pmod P$; $\chi(P)=1$ otherwise.

Since $J_n(\omega)\neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.

We have

$$\pi_2(N,3)\sim\frac{J_3(\omega)\omega}{2P_0\Phi^3(\omega)}\frac{N^2}{\log^3 N}.$$

The Polynomial $P_1^n+(P_2+1)^2$ Captures Its Primes [14]

(1)Let $n=4$, We have

$$P_3=P_1^4+(P_2+1)^2,$$

We have Jiang function

$$J_3(\omega)=\prod_{3\le P}(P^2-3P+3-\chi(P))\neq 0,$$

Where $\chi(P)=P$ if $P\equiv 1 \pmod 4$; $\chi(P)=P-4$ if $P\equiv 1 \pmod 8$; $\chi(P)=-P+2$ otherwise.

Since $J_n(\omega)\neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.

We have the best asymptotic formula

$$\pi_2(N,3)=\left|\{P_1,P_2:P_1,P_2\le N,P_1^4+(P_2+1)^2=P_3\text{ prime}\}\right|\sim\frac{J_3(\omega)\omega}{8\Phi^3(\omega)}\frac{N^2}{\log^3 N}.$$

It is the simplest theorem which is called Friedlander-Iwaniec problem [16].

(2)Let $n = 4m$, We have

$$P_3 = P_1^{4m} + (P_2 + 1)^2,$$

where $m = 1, 2, 3, \cdots$.

We have Jiang function

$$J_3(\omega) = \prod_{3 \le P \le P_i} (P^2 - 3P + 3 - \chi(P)) \ne 0,$$

where $\chi(P) = P - 4m$ if $8m \mid (P-1)$; $\chi(P) = P - 4$ if $8 \mid (P-1)$; $\chi(P) = P$ if $4 \mid (P-1)$; $\chi(P) = -P + 2$ otherwise.
Since $J_3(\omega) \ne 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime. It is a generalization of Euler proof for the existence of infinitely many primes.

We have the best asymptotic formula

$$\pi_2(N, 3) \sim \frac{J_3(\omega)\omega}{8m\Phi^3(\omega)} \frac{N^2}{\log^3 N}.$$

(3)Let $n = 2b$. We have

$$P_3 = P_1^{2b} + (P_2 + 1)^2,$$

where b is an odd.

We have Jiang function

$$J_3(\omega) = \prod_{3 \le P} (P^2 - 3P + 3 - \chi(P)) \ne 0,$$

Where $\chi(P) = P - 2b$ if $4b \mid (P-1)$; $\chi(P) = P - 2$ if $4 \mid (P-1)$; $\chi(P) = -P + 2$ otherwise.

We have the best asymptotic formula

$$\pi_2(N, 3) \sim \frac{J_3(\omega)\omega}{4b\Phi^3(\omega)} \frac{N^2}{\log^3 N}.$$

(4)Let $n = P_0$, We have

$$P_3 = P_1^{P_0} + (P_2 + 1)^2 \cdot$$

where P_0 is an odd. Prime.

we have Jiang function

$$J_3(\omega) = \prod_{3 \le P} (P^2 - 3P + 3 - \chi(P)) \ne 0,$$

where $\chi(P) = P_0 + 1$ if $P_0 \mid (P-1)$; $\chi(P) = 0$ otherwise.

Since $J_3(\omega) \ne 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime.
We have the best asymptotic formula

$$\pi_2(N, 3) \sim \frac{J_3(\omega)\omega}{2P_0\Phi^3(\omega)} \frac{N^2}{\log^3 N}.$$

The Jiang function $J_n(\omega)$ is closely related to the prime distribution. Using $J_n(\omega)$ we are able to tackle almost all prime problems in the prime distributions.

REFERENCES

[1] García, J.I., Sepúlveda, S. and Noriega-Hoces, L. (2010) Beneficial Effect of Reduced Oxygen Concentration with Transfer of Blastocysts in IVF Patients Older than 40 Years Old. Health, 2, 1010-1017.

[2] B. Riemann, Uber die Anzahl der Primzahlen under einer gegebener Grösse, Monatsber Akad. Berlin, 671-680 (1859).

[3] P. Bormein, S. Choi, B. Rooney, The Riemann hypothesis, pp28-30, Springer-Verlag, 2007.

[4] Chun-Xuan Jiang, Disproofs of Riemann hypothesis, Algebras Groups and Geometries 22, 123-136 (2005). http://www.i-b-r.org/docs/JiangRiemann.pdf.

[5] Tribikram Pati, The Riemann hypothesis, arxiv: math/0703367v2, 19 Mar. 2007.

[6] Laurent Schadeck, Private communication. Nov. 5. 2007.

[7] Laurent Schadeck, Remarques sur quelques tentatives de demonstration Originales de l'Hypothèse de Riemann et sur la possiblilité De les prolonger vers une théorie des nombres premiers consistante, unpublished, 2007.

[8] Chun-Xuan Jiang, Foundations of Santilli's isonumber theory with applications to new cryptograms, Fermat's theorem and Goldbach's conjecture, Inter. Acad. Press, 2002. MR2004c: 11001, http://www.i-b-r.org/Jiang. pdf.

[9] Chun-xuan Jiang, The simplest proofs of both arbitrarily long arithmetic progressions of primes, Preprint (2006).

[10] B. Kra, The Green-Tao theorem on arithmetic progressions in the primes: an ergodic point of view, Bull. Am. Math. Soc. 43, 3-23(2006).

[11] B. Green and T. Tao, The primes contain arbitrarily long arithmetic progressions. To appear, Ann. Math.

[12] T. Tao, The dichotomy between structure and randomness, arithmetic progressions, and the primes. In proceedings of the international congress of mathematicians (Madrid. 2006). Europ. Math, Soc. Vol.1, 581-609, 2007.

[13] A. Wiles, Modular elliptic curves and Fermat's last theorem, Ann. Math. 141, 443-551 (1995).

[14] Chun-Xuan Jiang, Fermat' s marvelous proofs for Femart' s last theorem, preprint (2007), submit to Ann. Math.

[15] Chun-Xuan Jiang, Prime theorem in Santilli's isonumber theory (II), Algebras Groups and Geometries 20, 149-170(2003).

[16] D.R. Heath-Brown, Primes represented by x^3+2y^3. Acta Math. 186, 1-84(2001).

[17] J. Friedlander and H. Iwaniec, The polynomial x^2+y^4 captures its primes. Ann. Math. 148, 945-1040(1998)

[18] Castro, C. , Granik, A. , & Mahecha, J. . (2001). On susy-qm, fractal strings and steps towards a proof of the riemann hypothesis. Physics.

[19] Zimmer, H. G. . (1971). An elementary proof of the riemann hypothesis for an elliptic curve over a finite field. Pacific Journal of Mathematics, 36(1).

[20] Shi, K. . (2003). A geometric proof of riemann hypothesis. Mathematics.

[21] Eswaran, K. . (2018). A rigorous proof of the riemann hypothesis from first principles.

[22] Huang, S. . (2008). Modeling the creative process of the mind by prime numbers and a simple proof of the riemann hypothesis. Mathematics.

[23] Mackay, R. S. . (2017). Towards a spectral proof of riemann's hypothesis.

[24] Garc, A. , & Doz, S. . (2013). A potential elementary proof of the riemann hypothesis.

[25] Xiang-Hu, M. A. . (2017). Ingenious proof of zeros distribution of the zeta function in riemann hypothesis. Sci-tech Innovation and Productivity.

[26] Botsko, M. W. . (2003). A simple proof of the derivative of the indefinite riemann-complete integral. Real analysis exchange.

[27] Shinya, H. . On the proof of the Riemann Hypothesis and multiplicities of the nontrivial zeros of the Riemann ζ-function.

[28] Destefano, Z. . A RIGOROUS PROOF OF THE ARC LENGTH AND LINE INTEGRAL FORMULA USING THE RIEMANN INTEGRAL.

[29] Ho, M. K. M. . (2014). A condensed proof of the differential grothendieck-riemann-roch theorem. Proceedings of the American Mathematical Society, 142(6).

[30] Anthony, A. M. M. . A SIMPLE PROOF OF THE RIEMANN HYPOTHESIS ?.

[31] Feng, N. , Wang, Y. , & Wu, R. . The Riemann Hypothesis and the possible proof.

[32] Dutt, P. . (1986). A riemann solver based on a global existence proof for the riemann problem.

[33] Oscar García-Prada. (1994). A direct existence proof for the vortex equations over a compact riemann surface. Bulletin of the London Mathematical Society, 26(1), 88-96.